Logit mixed logit under asymmetry and multimodality of WTP: a Monte Carlo evaluation

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Abstract

The logit-mixed logit (LML) model advances choice modelling by generalizing previous parametric and semi-nonparametric specifications and allowing retrieval of flexible taste distributions. Using standard operating conditions in the field, we report results from Monte Carlo experiments designed to assess the finite sample bias-variance tradeoff for the LML using as a benchmark conventional Mixed logit models (MXL) under asymmetric and multi-modal taste distributions. The LML specification always outperforms the MXL in terms of bias, but when the variance around modes is high the mean squared error (MSE) is lower than that of MXL only at sample sizes larger than usual and with some nuances. *D*-error minimizing experimental design predicated on multinomial logit significantly reduces MSE, but no clear winner is found between polynomial, step, and spline functions for the multidimensional grid function. Analysis of empirical data from a choice experiment on tap water shows that multimodality emerges only if higher number of node parameters are used in the LML.

Key words: logit mixed logit, choice modelling, random utility, utility in WTP-space, semi-parametric choice models

JEL classification: C14, C35, Q25

The modelling of how taste differs across people dominates the field of contemporary choice analysis. For the most part, to model such diversity, empirical studies rely on continuous taste distributions with a single modal value (normal, log-normal, triangular, etc.). Few applications account for the effects of multiple modal values (i.e. multimodality). Yet, accurately identifying high frequencies of taste over specific ranges can be of great practical importance in policy design. For example, when developing policies relies on sorting or price-discrimination (Chen and Iyer 2002; Belleflamme, Lam, and Vergote 2017) one can identify willingness to pay ranges with high frequency of people. Some degree of approximation to multimodal preferences can be achieved by using discrete, rather than continuous distributions, as implemented in latent class models. But this comes at the cost of 'lumpy' rather than 'smooth' distributions, which might be counter-intuitive in other respects. Real taste distributions not only are likely to be continuous and to display more than one modal value, but they are also often asymmetric around these modal values. This is corroborated by the few empirical studies that addressed this issue in transport choice (Fosgerau and Hess 2007), choice of video streaming services (Train 2016) and in food choice (Scarpa, Thiene, and Marangon 2008; Caputo et al. 2018; Bazzani, Palma, and Nayga 2018). In this study we first report the results of a large scale Monte Carlo (MC) esperiment to explore the properties of a recently introduced semi-parametric estimator capable of uncovering multimodality and asymmetry of continuous taste distributions: the logit mixed logit model Train (2016). Secondly, we provide an empirical application on choice experiment data whose results demonstrate the practical effectiveness of the proposed specification.

Specifically, we explore the properties of the logit mixed logit (LML) specification using as a benchmark the conventional mixed logit with normal random coefficients (MXL), which has emerged as the default choice in most published applications. In our literature review of five top journals in environmental economics we find that in the period 2012-2019 as many as 89 papers used MXL specifications with normal distributions for the random taste parameters. Both MXL and LML estimators are asymptotically consistent under the correct specification. So, our MC design is geared towards increasing our understanding of their bias-variance tradeoff in finite sample sizes. This knowledge is of practical importance to assess the conditions for asymmetry and multimodality to be effectively addressed.

As the name suggests, the LML contains two logit formulations: one for the decision maker's probability to choose an alternative, the other for her/his probability of having given taste parameter values from a specific interval. The exponential terms in the latter logit formulation ensure a positive probability, while the denominator ensures normalization (i.e. that all probabilities sum to one). The shape of the logarithm of the mixing distribution can be defined by different type of functions such as polynomials, step functions, and splines, among many others. This estimator has been supplied with general purpose code in MatLab and presents very desirable computational features (see Bansal, Daziano, and Achtnicht 2018b, for further refinements). Early applications of the LML involved a favourable comparison between stated and revealed preference choice data in an experimental setting using induced value (Bazzani, Palma, and Nayga 2018), and an exploration of the consequences of range size, asymmetry and multimodality when the assumption of taste distribution is normal in a stated food choice (steak) setting (Caputo et al. 2018).

Mixed logit model estimators, such as MXL and LML, are consistent only asymptotically and under the correct specification. When the true taste distribution is multimodal and asymmetric, if these features are ignored—as it happens with MXL based on continuous unimodal distributions—consistency is lost. This is not the case for the LML estimator, for which—as the sample size increases—we obtain convergence in probability to the true parameter values. At finite sample sizes, however, both are biased, albeit with different variances. One contribution of this study is to characterize the bias-variance tradeoff within a practical range of sample sizes, under the specific conditions of multimodality and asymmetry assumed for the data generating processes (DGPs). To explore such tradeoff we use the mean squared error decomposition:

$$(1) \quad MSE(\hat{\theta}) = BIAS(\hat{\theta})^2 + Var(\hat{\theta}),$$

derived from a specifically designed set of MC experiments, based on 2 DGPs each repeated at 5 sample sizes, 3 separate experimental designs, and 2 lengths of choice sequences (60 experiments in total) each generating 1,000 synthetic datasets, which in turn were estimated using 14 specifications (see the original working paper by Scarpa, Franceschinis, and Thiene 2017, for additional details). The total number of estimates obtained are 840,000, for a total of 840 empirical measures of $MSE(\hat{\theta})$.

With these experiments we also explore the effects of other practically salient determinants of MSE, such as experimental design criteria, type of function for the grid of probability weights and length of the sequence of choices. While working at this project we came across the study by Bansal, Daziano, and Achtnicht (2018a), who conducted a similar MC study, albeit at a much smaller scale of resolution than ours,[1] and studied the conditions that determine the ability of LML to retrieve random coefficient distributions based on frequently employed parametric distributions (normal, log-normal, uniform, symmetric bimodal normal, uniform, discrete and discrete log-normal) using specifications with utility in WTP-space. One limitation of their choice of DGPs is that they only explored symmetric bimodal distributions with identical and rather small variances around the modal values (e.g. bimodal normal with same variance and means -1 and 1; discrete with probability 1/3 and mass at $-2, 0$, and 2). In real life multimodal distributions are more likely to be asymmetric, and there is often evidence of more than two modal values each with a diverse and possibly large variance. Asymmetry implies differences in variances around the modal values. When variance is large around one of the modal values and these values are close, there is an obvious issue of identification of modal values, and consequently accurate estimation is complicated.

While we contribute to the literature by studying the complementary issue of asymmetry of distributions, our study is also an extension of their work, which did not place any specific focus on issues such as the bias-efficiency tradeoff at practical sample sizes, the presence of a third modal value in the data generatig process, and on the role of efficient experimental design, which are all additional contributions of this study. Together the two studies will provide a rather complete characterization of the finite sample size properties of the logit-mixed logit model.

The correct retrieval of modes in preference distribution is salient in applied welfare analysis for public goods for their relation to median voter behaviour, and hence political markets for public good provision (e.g. see the discussion in Mitchell and Carson 1989). Finally, in this study we endeavour to refer to the common operating conditions prevailing in the agricultural, food and environmental economics literature, which are quite different from those prevailing in transport choice analysis, which instead inspired Bansal, Daziano, and Achtnicht (2018a).

In addition to the MC experiment results, we provide empirical saliency by illustrating a case study in which standard parametric approaches lead to overlooking some features that instead emerge as important once the LML estimator is employed. We analyze the preferences of 832 households in a part of the province of Vicenza (North Italy) for tap water attributes. Residential water supply is a complex quasi-public good jointly managed by water utilities and regulatory bodies (Willis and Scarpa 2002; Willis, Scarpa, and Acutt 2005; Hensher, Shore, and Train 2005; Scarpa, Willis, and Acutt 2007; Rungie, Scarpa, and Thiene 2014; Thiene, Scarpa, and Louviere 2015) as natural monopolies. Gathering information about customer preference is important in order to strategically define investment in infrastructure to improve factor services, such as water delivery, quality of water treatment and sewer services. If a water factor service produces benefits to utility costumers, this is deemed worth of further investment by improving infrastructure and it may strengthen the case for increasing water tariffs in the eyes of regulators.

Exploring the benefits of semi-parametric LML specifications over standard parametric ones in this empirical context is worthwhile. Although there is insufficient space to delve deep into this here, knowing the distribution features of the benefits can be important to calibrate infrastructure investment. For example, knowing if the high benefit mean of the entire distribution is underpinned by two modes, with one at a relatively low benefit value and a second at a high level of benefits, has dramatically different implications from a situation in which the population displays a single mode perhaps centred on the mean, as it would induce different strategies in investments and funding. The objective of this empirical application is to explore the implications of alternative LML specifications with varying number of parameters on the estimates of the distributions of WTP values for the improvement of tap water services. Since the true distribution of WTP is unknown, we compare the distributions of WTP estimates of LML with MXL and assess the benefits of using LML over parametric specifications. Asymmetry and multimodality emerge as key features.

The remaining paper is organized as follows: the next section illustrates MXL and LML models, the subsequent section describes the MC experiment design and discusses simulation results. The empirical study and its results are described in the section preceding the conclusions of the paper.

Econometric modeling

The repeated choice Mixed Logit Model with normals (MXL)

We start with the illustration of the most commonly used mixed logit specification to date. The repeated choice MXL model represents random taste heterogeneity by allowing for different preference parameters for each decision-maker (Revelt and Train 1998). Conditional on the individual's taste coefficients $\boldsymbol{\beta}_n$, the utility derived by individual n from choosing alternative i in choice occasion t is logit:

(2) $$U_{nit} = \boldsymbol{\beta}'_n \mathbf{x}_{nit} + \varepsilon_{nit}, \text{ where } n = 1,...,N; i \in J; t = 1,...,T.$$

$\boldsymbol{\beta}_n$ is a vector of parameters that varies across individuals with an assumed continuous mixing distribution in the population; $\mathbf{x}_{njt}$ is a conformable column vector of observed attributes of alternative i; ε_{njt} is the independent error term assumed to follow a Gumbel distribution. The conditional probability $P_n(it|\boldsymbol{\beta}_n)$ of individual n choosing alternative i in choice occasion t is logit:

(3) $$P_n(it|\boldsymbol{\beta}_n) = \frac{\exp(\boldsymbol{\beta}'_n \mathbf{x}_{nit})}{\sum_{j=1}^{J} \exp(\boldsymbol{\beta}'_n \mathbf{x}_{njt})}.$$

Many variants of the MXL models can be obtained by assuming different mixing distributions of the random parameters. The most commonly used is the MXL that imposes a multivariate normal mixing distribution, i.e., $\boldsymbol{\beta}_n \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Let $y_{nit} = 1$ if individual i chooses alternative i in choice situation t, and 0 otherwise. For a panel of T choices, the unconditional probability of the sequence of T preferred alternatives when individual n is facing J alternatives in each choice task is:

(4) $$P_n(iT|\boldsymbol{\beta}, \boldsymbol{\Sigma}) = \int \left\{ \prod_{t=1}^{T} \prod_{j=1}^{J} \left[\frac{\exp(\boldsymbol{\beta}'_n \mathbf{x}_{nit})}{\sum_{j=1}^{J} \exp(\boldsymbol{\beta}'_n \mathbf{x}_{njt})} \right]^{y_{nit}} \right\} \phi(\boldsymbol{\beta}_n | \boldsymbol{\mu}, \boldsymbol{\Sigma}) d\boldsymbol{\beta}_n,$$

where $\phi(\boldsymbol{\beta}_n|\boldsymbol{\mu}, \boldsymbol{\Sigma})$ is the multivariate normal density function with mean hyperparameter vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$ for the random taste parameters $\boldsymbol{\beta}_n$. Hyperparameters in the MXL model are typically estimated via maximum simulated likelihood (Gouríeroux and Monfont 1996).

The repeated choice Logit Mixed Logit model (LML)

In LML models (Train 2016), the joint mixing distribution of the random parameters $\boldsymbol{\beta}_n$ is assumed to be discrete over a finite support set S. Discretization is not a constraint because the support set is essentially a multidimensional grid. The analyst can choose this to be made larger and denser by considering a broader domain of parameters and a higher number of grid points. The joint probability mass function of random parameters in LML is specified by the following logit formula:

$$(5) \quad w_n(\boldsymbol{\beta}_r|\boldsymbol{\alpha}) = \Pr(\boldsymbol{\beta}_n = \boldsymbol{\beta}_r) = \frac{\exp\left(\boldsymbol{\alpha}'\mathbf{z}(\boldsymbol{\beta}_r)\right)}{\sum_{s\in S}\exp\left(\boldsymbol{\alpha}'\mathbf{z}(\boldsymbol{\beta}_r)\right)},$$

where $\boldsymbol{\alpha}$ is a vector of parameters, $z(\boldsymbol{\beta}_r)$ defines the shape of the mixing distribution, and r denotes the point in the grid for the evaluation of $\boldsymbol{\beta}$. The unconditional probability of the sequence of choices of individual n is the following weighted sum:

$$(6) \quad P_n(jT|\boldsymbol{\alpha}) = \sum_{r\in S}\left\{\prod_{t=1}^{T}\prod_{j=1}^{J}\left[\frac{\exp(\boldsymbol{\beta}_n'\mathbf{x}_{nit})}{\sum_{k=1}^{J}\exp(\boldsymbol{\beta}_n'\mathbf{x}_{njt})}\right]^{y_{nit}}\right\} w_n(\boldsymbol{\beta}_r|\boldsymbol{\alpha}).$$

In LML models, the vector $\boldsymbol{\alpha}$ is estimated using the (simulated) maximum likelihood estimation procedure. This obviates the frequent problem of a lengthy convergence time and testing of stability of posterior, typical of Bayesian approaches, which is often a hindrance in panel choice models.[2] Inclusion of all the points of the support set in the estimation of LML is unnecessary and computationally expensive, so a subset of points is drawn from S. Using the logit formula in equation 5 to compute probability mass of random parameters results into an efficient computation of the gradient of the sample log-likelihood, facilitating the use of gradient-based methods in estimation.

The z functions in LML models

A critical issue in LML model is the specification of the S variables that describe the mixing distribution and its grid points. Following Train (2016), we adopt three different functions: *i*) polynomials (LML-Poly), *ii*) step function (LML-Step function), *iii*) spline (LML-Spline).

An important feature of LML-Poly is that many commonly employed distributions can be approximated by varying the order of the polynomial. For example, Train (2016) shows that the normal distribution can be introduced in LML framework by considering $z(\boldsymbol{\beta}_r)$ to be a second order polynomial of a special form. The polynomial can be extended to higher orders to gain greater flexibility of the mixing distribution, bearing in mind that the number of inflection points is equal to the polynomial order minus one. Among the various categories of polynomials, orthogonal polynomials (e.g. Legendre, Hermite, Jacobi, Chebyshev, Bernstein etc.) have the advantage of having uncorrelated terms. Dependence among the elements of multidimensional $\boldsymbol{\beta}$ can still be captured by cross-products of the terms of each element's polynomial.

A second alternative consists of defining $z(\boldsymbol{\beta}_r)$ as a step function based on a grid over the parameter ranges (i.e. the support set S). Suppose S is partitioned into M subsets, labelled as T_m where $m = \{1, 2, ..., M\}$. Let the probability mass function $W(\boldsymbol{\beta})$ be the same for all points within each subset, but different among subsets. Then, the logit formula for the probability masses is:

$$(7) \qquad w_n(\boldsymbol{\beta}_r|\boldsymbol{\alpha}) = \Pr(\boldsymbol{\beta}_n = \boldsymbol{\beta}_r) = \frac{\exp\left(\sum_{m=1}^{M} \alpha_m I(\boldsymbol{\beta}_r \in T_m)\right)}{\sum_{s\in S}\exp\left(\sum_{m=1}^{M} \alpha_m I(\boldsymbol{\beta}_s \in T_m)\right)}.$$

The z variables are the M indicators which identify the subset containing $\boldsymbol{\beta}_r$. If the subsets do not overlap, then one of the coefficients is normalized to zero. With overlapping subsets, instead, one coefficient is normalized to zero for each possible way of covering the set S.

In LML-Step function the number of estimated parameters is equal to the number of grid points.

Finally, a linear spline can be used to define $\mathbf{z}(\beta)$, once defined over h knots. Spline functions connect piece-wise polynomial functions at a high degree of smoothness and in a linear setting they can be written in the form $\boldsymbol{\alpha}'\mathbf{z}(\beta)$, as needed in the LML specification. Consider a simple example of spline with $h = 2$ and with starting point at β_1, ending point in β_4, and place the two knots at β_2 and β_3, with $\beta_1 < \beta_2 < \beta_3 < \beta_4$. Let the corresponding elements of the vector $\boldsymbol{\alpha}$ define the spline heights. The elements of vector $\mathbf{z}(\beta)$ in this case are:

$$
\begin{cases}
z_1(\beta) = \left(1 - \frac{\beta-\bar{\beta}_1}{\bar{\beta}_2-\bar{\beta}_1}\right) I(\beta \leq \bar{\beta}_2) \\
z_2(\beta) = \left(\frac{\beta-\bar{\beta}_1}{\bar{\beta}_2-\bar{\beta}_1}\right) I(\beta \leq \bar{\beta}_2) + \left(1 - \frac{\beta-\bar{\beta}_2}{\bar{\beta}_3-\bar{\beta}_2}\right) I(\bar{\beta}_2 < \beta \leq \bar{\beta}_3) \\
z_3(\beta) = \left(\frac{\beta-\bar{\beta}_2}{\bar{\beta}_3-\bar{\beta}_2}\right) I(\bar{\beta}_2 < \beta \leq \bar{\beta}_3) + \left(1 - \frac{\beta-\bar{\beta}_3}{\bar{\beta}_4-\bar{\beta}_3}\right) I(\beta_3 < \beta) \\
z_4(\beta) = \left(\frac{\beta-\bar{\beta}_3}{\bar{\beta}_4-\bar{\beta}_3}\right) I(\beta_3 < \beta)
\end{cases}, \tag{8}
$$

where $I(\cdot)$ is an indicator function.

Monte Carlo experiment

To assess the performance of different model specifications, we conducted a Monte Carlo (MC) study based on a utility function with three attributes with random coefficients. The first and the second attribute are assumed to be non-monetary, whereas the third is the price attribute. Because the use of dummy-coding is prevalent in this literature, the two non-monetary attributes were coded as dummy variables, taking the values of 0 and 1, indicating their presence or absence in the alternative they describe. The price attribute was continuous and also with two levels, with values of 1 and 2. The true data generation processes (DGPs) were based on asymmetric and bi- and tri-modal distributions, with utility specified in WTP-space, so that coefficients are interpretable as marginal WTPs ($mWTP$).

Random utility specification

Consistently with random utility theory, it was assumed that respondent select the alternative with maximum utility out of two available alternatives. The utility of respondent n for alternative i in choice occasion t was specified in the WTP-space (Train and Weeks 2005) as:

$$(9) \quad U_{nit}(\beta_n) = \lambda_n^*(\omega_n^1 x_{nit}^1 + \omega_n^2 x_{nit}^2 - p_{nit}) + \varepsilon_{nit},$$

where λ_n^* is the price/scale coefficient and ω_n^1 and ω_n^2 are the $mWTP$ for attribute 1 and attribute 2, while ε_{nit} is distributed i.i.d. Gumbel.

Data generating processes

To compare performance between MXL and LML models at increasing levels of complexity of $mWTP$ distributions, we generate two DGPs. In DGP 1, ω_n^1 and ω_n^2 are generated following a bimodal distribution, obtained by mixing two normals, whereas the price/scale coefficient λ_n^* is assumed to follow a mixture of two log-normals, to ensure a positive sign. The price coefficient p_i was assumed to be fixed to -1. The random utility component ε_{njt} follows a standard Gumbel distribution, so as to have a logit choice probability. The distribution parameters in DGP 1 are asymmetric and bimodal, as follows:

$$(10) \quad \omega_n^1 \sim \mathcal{N}(\boldsymbol{\mu}^1, \boldsymbol{\Sigma}^1) \text{ with } \boldsymbol{\mu}^1 = \begin{bmatrix} 0.5 \\ 1.2 \end{bmatrix} \boldsymbol{\Sigma}^1 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.04 \end{bmatrix} \text{ with } \Pr = \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix},$$

$$\text{(11)} \quad \omega_n^2 \sim \mathcal{N}(\boldsymbol{\mu}^2, \boldsymbol{\Sigma}^2) \text{ with } \boldsymbol{\mu}^2 = \begin{bmatrix} -1.5 \\ 1.5 \end{bmatrix} \boldsymbol{\Sigma^2} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \text{ with } \Pr = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix},$$

$$\text{(12)} \quad \lambda_n^* = \exp(\theta), \theta \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ with } \boldsymbol{\mu} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} \boldsymbol{\Sigma} = \begin{bmatrix} 0.25 & 0 \\ 0 & 1.0 \end{bmatrix} \text{ with } \Pr = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

The shape of the distributions for both random $mWTP$s for attributes used in DGP 1, ω^1 and ω^2, are shown in the upper panel of Figure 1. Note that the distribution for ω^1 has two different modes, one at high benefits, the second at low benefits. The density for the latter is much higher than the density for higher benefits. This represents a situation with a small group of high beneficiaries and a much larger group of low beneficiaries, not uncommon in practice.

Also note that in the distribution for ω^2 one mode is negative and has higher density than its positive counterpart. This is to denote asymmetric distributions of winners and losers linked to the supply of that binary level attribute. This is often the case for attribute controversially valued by the population. In the real world, these two forms of asymmetric bimodality in the distribution of benefits are common (e.g. from a public good provision). If they were incorrectly assumed to have a single mode with intermediate modal value then severely erroneous policy prescriptions would follow.

In DGP 2, both $mWTP$s have asymmetric and trimodal distributions, obtained as a mixture of three normals (Figure 1, lower panel). The DGP 2 parameters have the following values:

$$\text{(13)}\quad \omega_n^1 \sim \mathcal{N}(\boldsymbol{\mu}^1, \boldsymbol{\Sigma}^1) \text{ with } \boldsymbol{\mu}^1 = \begin{bmatrix} 1.5 \\ 3.5 \\ -1.2 \end{bmatrix} \boldsymbol{\Sigma}^1 = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \text{ with } P = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix},$$

$$\text{(14)}\quad \omega_n^2 \sim \mathcal{N}(\boldsymbol{\mu}^2, \boldsymbol{\Sigma}^2) \text{ with } \boldsymbol{\mu}^2 = \begin{bmatrix} 5.5 \\ 3.0 \\ 1.2 \end{bmatrix} \boldsymbol{\Sigma}^2 = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.09 \end{bmatrix} \text{ with } P = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix},$$

$$\text{(15)}\quad \lambda_n^* = \exp(\theta), \theta \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \text{ with } \boldsymbol{\mu} = \begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix} \boldsymbol{\Sigma} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix} \text{ with } P = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}.$$

In this case, in the distribution for ω^1, the two positive modal densities (y-axis) differ by less than in DGP 1 and are contrasted by the highest modal density in the negative x-axis. For the value distribution of attribute 1 this denotes strong clustering of losers, and bimodal winners, but with small variance around the modal values.

The distribution for ω^2 has only positive modal values, but with three different modal densities, the highest of which is at low level of benefits, accompanied by two similar level densities at higher benefit levels. As for DGP 1, it is intuitive to conclude that these forms of asymmetric trimodality in the benefits distribution, in case they were erroneously assumed to be unimodal, will also lead to seriously sub-optimal policy actions.

Note also that in both DGPs ω^1 has much smaller variance values around the modes than ω^2. Larger variance around modal values of random coefficients is expected to require larger sample sizes to accurately disentangle the location of the respective modes. How much larger is a further question we investigate here.

Experiment features and error measures

We denote with h the generic synthetic dataset $h = 1, \ldots, H = 1{,}000$ generated in each of the 60 MC experiments, which are then used to estimate 14 model specifications, for a total of $R = 60 \times 14 = 840$ sets of estimation error measures denoted by $r = 1, \ldots, R = 840$. The 14 specifications consist of:

- one MXL in preference space with normal distributions for each non-price attribute,
- one MXL in WTP space with normal coefficients for all non-price attributes,
- four LML-Poly with varying number of parameters (12, 24, 36, 48),
- four LML-Step with varying number of steps (12, 24, 36, 48),
- and four LML-Spline with varying number of knots (12, 24, 36, 48).

All LML models are with utility in WTP-space and all price/scale λ^* coefficients are log-normal or mixture of log-normals. Data generation and all estimations were performed in MatLab using Train's code modified to fit our purpose. Choice probabilities are simulated in the sample log-likelihood with 250 Halton draws. To simulate the sampling distributions properties of $mWTP$ values from the MXL in preference space, $10{,}000$ draws were taken from the estimated distribution of each non-monetary attribute coefficient. Each draw is then divided by a draw from the estimated distribution of the cost coefficient. Standard statistics for the distribution of these WTPs were then calculated for these draws (but see the caveats in Daly, Hess, and Train (2012)). So, we evaluate the performance of each of the 14 specifications by computing the MSE and the $BIAS$ over the $h...H = 1,000$ synthetic samples in each of the 840 MC experiments:

$$(16) \quad MSE(\hat{\omega}) = \frac{1}{H}\sum_{h=1}^{H}(\hat{\omega}_h - \omega)^2, h = 1,\ldots,H;$$

$$(17) \quad BIAS(\hat{\omega}) = \frac{1}{H}\sum_{h=1}^{H}\hat{\omega}_h - \omega, h = 1,\ldots,H;$$

From the above, by using equation (1), we can derive the finite sample variance of each estimator as $VAR(\hat{\omega}) = MSE(\hat{\omega}) - [BIAS(\hat{\omega})]^2$. This allows us to identify the empirical bias-variance tradeoff under different MC experimental factors.

In the case of welfare estimates the sign of the bias is not immaterial as one might not worry about over-estimation (positive bias) and instead be concerned by under-estimation (negative bias). To gain insight and evaluate the relative departure from the true population values, in each experiment r we also compute the means of (a) relative absolute error ($MRAE$), (b) relative negative error ($MRNE$) and (c) relative positive error ($MRPE$) of the estimates. Hence we compute:

$$(18) \quad MRAE = \pi = \frac{1}{H}\sum_{h=1}^{H}\left|\frac{\hat{\omega}_h - \omega}{\omega}\right|;$$

$$(19) \quad MRNE = \pi^- = \left[\sum_{h=1}^{H} 1_h\left(\frac{\hat{\omega}_h - \omega}{\omega} < 0\right)\right]^{-1}\sum_{h=1}^{H}\left|\frac{\hat{\omega}_h - \omega}{\omega}\right| \times 1_h\left(\frac{\hat{\omega}_h - \omega}{\omega} < 0\right);$$

$$(20)\quad MRPE = \pi^{+} = \left[\sum_{r=1}^{H} 1_h\left(\frac{\hat{\omega}_h - \omega}{\omega} > 0\right)\right]^{-1} \sum_{h=1}^{H} \left|\frac{\hat{\omega}_h - \omega}{\omega}\right| \times 1_h\left(\frac{\hat{\omega}_h - \omega}{\omega} > 0\right);$$

where $1(\cdot)$ is an indicator function, ω is the $mWTP$ value used in the DGP and $\hat{\omega}_h$ is the value estimated from the h^{th} synthetic dataset, $h = 1, \ldots, H$.

In revealed preference data studies researchers do not exercise control over the allocation of attribute values across alternatives, but in most stated choice applications this is the outcome of an error-optimized experimental design (Sándor and Wedel 2001; Ferrini and Scarpa 2007; Rose and Bliemer 2009), and an adequate choice of design might afford significant efficiency gains in $mWTP$ estimation. It is unclear if efficient design can also reduce estimation error for the LML model. We investigate this here. In both DGPs we use three experimental designs: *i*) *D*-error minimizing design (we call this *D*-efficient), *ii*) random design, *iii*) full factorial design.

In order to evaluate the effect of the length of choice sequence on models performance, we generate two different sets of data: the first is built assuming that each respondent faces four choice tasks, the second assuming eight choice tasks. These panel lengths are common, for examples, in food and environment stated choice experiments.

Similarly, to investigate the effect of the sample size, we generate five panel datasets with increasing number of simulated panels (*N*): 70, 210, 490, 980, 1960. This allows us to investigate the role of sample size on performance of the LML model. Sample sizes are defined so as to have the same number of respondents for each block of the full fractional design.

Bias versus efficiency tradeoffs

Figure 2 illustrates the relative contributions of $BIAS^2$ and variance to the MSE at different sample sizes for the three estimators: MXL-P with utility in preference space (fixed price coefficient), MXL-W with utility in WTP space, and LML. Note that at each sample size, the bias is always smaller for the LML, but its variance is much higher than the MXL at small sample sizes ($N = 210$). This suggests that to reap the benefits of the LML practitioners need to employ large sample sizes. However for ω^1—the coefficient with low variance around modal values in both DGPs—, already at a sample size of $N = 490$ LML has a variance component of the MSE, which is low enough to outperform (or do as well as) the MXL models. This happens both in the bimodal and the trimodal case for ω^1.

When the DGP has a high variance around modal values, (as for ω^2), the LML outperforms the MXL at a sample size in excess of about $N = 1000$ respondents when its distribution is trimodal. However, when its distribution is bimodal, already at $N = 490$ the LML outperforms the MXL-P, but not the MXL-W.

This suggests that some prior knowledge of the variance around modal values, and of the number of modes may inform practitioners of the type of estimator to use: if such variance is small and modal values are few, then the LML can be effective at relatively smaller sample sizes than in opposite situations. This result will need to be confirmed with further investigations, but it appears reasonable.

Response surface models

Focussing on the subsets of 720 error estimates involving the LML estimator, we are interested in how the error indicators of each experiment in equations (1-20) depend on the g factors in the MC experiment, which are:

i) sample size (N),

ii) number of parameters (κ),

iii) type of $z(\cdot)$ function (we used step function as the baseline),

iv) type of experimental design (full factorial and random designs were used as baseline),

v) number of choice tasks per respondent t, and

vi) DGP (trimodal was used as baseline).

We also examine interaction terms between each of the above factors, with the exception of those terms involving the $z(\cdot)$ function, as they are statistically insignificant.

Let the g factors determining the error determinants for error estimate r be denoted by $\mathbf{s}_r$. In order to succinctly report and discuss such effects we use two types of response surface models, (*i*) an OLS regression for when the dependent variable y_r^* is continuous (i.e. for MSE), and (*ii*) a fractional response logit (FRLGT), both reporting standard errors clustered by MC experiment (Papke and Wooldridge 1996; Wooldridge 2011) when the dependent variable is a fraction π_r (i.e. for $MARE = \pi_r^*, MNRE = \pi_r^-$ and $MPRE = \pi_r^+$).

The two models give rise to two different marginal effects on the outcome of each error estimate r:

$$(21) \quad y_r^* = \boldsymbol{\delta}'\mathbf{s}_r + \varepsilon_r \rightarrow \frac{\partial y_r^*}{\partial s_g} = \delta_g$$

$$(22) \quad \pi_r^{\cdot} = \Lambda(\boldsymbol{\delta}'\mathbf{s}_r) \rightarrow \frac{\partial \pi_r^{\cdot}}{\partial s_g} = \delta_g \Lambda(\boldsymbol{\delta}'\mathbf{s}_r)(1 - \Lambda(\boldsymbol{\delta}'\mathbf{s}_r)),$$

where $\Lambda(\boldsymbol{\delta}'\mathbf{s}_r) = \left[1 + \exp(-\boldsymbol{\delta}'\mathbf{s}_r)\right]^{-1}$. The full set of results (some of which are in the online appendix) is available from the authors upon request. However, Williams (2009) showed that the use of interaction terms is potentially problematic with nonlinear models such as logit and probit. So, for the FRGLT model we report only the main marginal effects and ignore interactions.

Determinants of MSE and MRAE

Table 1 reports marginal effects of determinants from both OLS and FRLGT models for the *MSE* (left part of the table) and the *MRAE* (right part). The table reports results for ω^1, but similar results were obtained for ω^2 and are available from the authors.

In the OLS model the only insignificant variables are the types of $z(\cdot)$ functions. We conclude that polynomial, spline or step functions are equivalent in estimation error for LML. All other variables display the expected negative signs and are significant in their main effects. The magnitudes of the marginal effects on the *MSE* demonstrate that one extra parameter in κ has the same effect as using a D-error minimizing efficient design, while doubling the choice tasks from 4 to 8 increases accuracy by little less.

The significance and signs of interaction effects tell us that (*i*) the effect of larger $\sqrt{N}$ increases with an extra κ, (*ii*) but is diminishes for bimodal DGPs, and (*iii*) that an extra κ also has a smaller effect for bimodal DGPs.

In the FRLGT model we basically obtain the same results for the $MRAE$. The marginal effects (timed by 100) have a more intuitive explanation in this case. The strongest effect is shown for the D-error minimizing design, followed by a longer choice tasks sequence, while one unit increase in $\sqrt{N}$ has the same effect as having an underlying bimodal, rather than trimodal DGP. Note that using an efficient design produces nearly twice the efficiency impact of a one unit of $\sqrt{N}$, even though this is derived under parametric logit assumptions. This is also potentially valuable to researchers that can focus on good design and longer sequences, rather than increase sample size.

Figure 3 illustrate the effects of MC factors on the *MRAE* by means of kernel densities across all experiments (unconditional). While sample size effects are obviously the strongest, it is worth noting that the D-error minimizing design predicated on the MNL model, in this context has an effect that clearly trades off error with efficiency: if one is happy to accept an expected error in the 10-15 percent range, the random or full factorial designs deliver this with good likelihood. The D-error minimizing design affords both higher likelihood values of *MRAE* lower than 8 percent, but also at values higher than 15 percent, where other designs are have low desnities.

Determinants of negative and positive error in ω^1

In table 2 we report OLS and FRLGT regressions that explore the differential effect for positive and negative errors in estimation. The left most column reports the OLS estimates for the linear probability model explaining the variation in the pooled (stacked) sample of *MNRE* (under-predictions) and *MPRE* (over-predictions), or π_r. The OLS regression in the second column, in which the dummy variable for under prediction and its interaction terms with $\sqrt{N}$ are dropped, has a markedly lower $\bar{R}^2$, showing that over- and under-estimates have different linear projections. A formal Chow test shows that only the coefficients for $\sqrt{N}$ and for the dummy for negative errors significantly differ across the two sub-samples.

The third and fourth columns report OLS estimates of different linear probability models to the two sub-samples and show how the marginal effects differ across. These effects are also reported for the FRLGT in the right most columns, for comparison. We note that negative relative errors are lower on average (as demonstrated by the significant dummy coefficient) and that sample size increases are more effective in reducing the positive relative errors than the negative ones. The magnitude of marginal effects of other significant determinants are also stronger for other factors (e.g. D-efficient, $T = 8$) in reducing positive errors, but the difference is insignificant at this sample size. However, this might be a consequence of our specific choice of DGPs.

Considering that this model is estimated with a random utility specification in the WTP space, these results are particularly instructive in those applied contexts in which over estimates of $mWTP$s are likely to occur. The following empirical case study provides an example in such a respect.

An empirical application: WTP for tap water

To add saliency to the Monte Carlo results, we apply the LML estimator to data from an empirical application based on a discrete choice experiment (DCE) developed to elicit households' preferences for tap water attributes in the province of Vicenza (northern Italy). The area under investigation is known as a tannery district. In fact, it is the most important district of that type in Italy and one of the most important in Europe, as it accounts for nearly one third of fine European leather production (UNIC 2010).

The leather industry is a potential big polluter, due to the large amount of water required to treat hides, which are preserved using salt during their transport from South America or other far away origins. Consequently, wastewater from hides treatment plant, when improperly treated may affect freshwater quality in the area.

Historically this industry was located at the foothills of the Alps and it prospered here because of the several artesian springs providing a regular flow of one of the most pristine water sources in Italy, which was immediately put to a very polluting use. Water pollutants are present in low concentrations in hides, but may have high toxicity as tanning processes make use of toxic heavy metals like chrome and other chemical pollutants (e.g., sulphate and sodium chloride).

The current water charging system for public waste water processing is based on threshold concentrations of contaminants per unit of volume of water used, rather than on total discharged load of contaminants. Hence, large amounts of pristine water from local springs are used to dilute concentrations of industrial pollutants. To give a sense of proportion, the capacity of the local sewage plant is sufficient for a population of 1.5 million, while the local population is only about 115,000 residents. Thus, information about householders' preferences for tap water attributes is crucial for local authorities in order to strategically set water tariffs and plan investments in infrastructure.

Much of the necessary infrastructure for industrial water treatment would otherwise benefit tanneries, which would then be heavily subsidized by residential water users, causing a major misallocation of resources.

The DCE was based on five water quality attributes, namely:

i) the frequency with which chlorine odor can be smelled in water use (daily, once a week, once a month, never or always),

ii) the frequency with which chlorine taste could be tasted in the water (same frequencies as for odor),

iii) turbidity due to fine air bubbles (absent, low, medium or high turbidity),

iv) calcium carbonate staining in pipes (presence/absence of staining), and *v*) the cost attribute, which was described as the additional yearly amount of money that a household would pay (in water bills) at current consumption levels.

The experimental design adopted in the study was based on the criterion of Bayesian D-error minimization where the error was computed at parameter estimates obtained from a preliminary prior study of 80 households based on an initial design orthogonal on the differences. The point estimates from the pilot study informed the prior distribution for the Bayesian design, and the standard errors defined the variances of the prior distributions, which were assumed normal. Probabilities were derived from a simulation based on 200 Halton draws, and used to construct a final design using Ngene (ChoiceMetrics 2009). The design resulted in 36 choice tasks, and was blocked into four orthogonal blocks of nine choice tasks each.

Using the datasets obtained with the CE survey from a sample of 832 respondents (Thiene, Scarpa, and Louviere 2015), we estimated the 14 model specifications previously listed and we added two latent class models with respectively two and three classes. These capture perfectly correlated multimodality, but ignore variance around modal values.

To compare performance across models with different number of parameters we report in table 3 the simulated log-likelihood at convergence ($\mathscr{L}^*$), along with the Akaike information criteria (AIC) and the Bayesian information criteria (BIC). Given the importance of multimodality in this context, we also report the number of modal values of the estimated distributions of random coefficients ($mWTP$).

Based on the Monte Carlo results and the large number of observations in our water preference dataset, we expect the LML models to outperform the MXL ones in terms of fit to the data. We also expect that LML specifications with large number of parameters κ outperform those with fewer parameters. Finally, we expect LML specifications (especially those with large number of parameters) to be able to retrieve the features of real underlying distributions, even when these are asymmetric and multimodal.

Model fit and estimated modes

All the information criteria in table 3 favor LML specifications, as compared to the MXL and LC specifications. The results also support the MC experiment finding of an increase of model performance at large κ at this sample size of $N = 832$.

In terms of performance across different z functions within LML, the LML-Spline specification emerges as the best when based on $\kappa = 55$, according to the AIC, but when based on $\kappa = 44$ according to the BIC, which is unsurprising as this criterion applies a heavier penalty on over-parameterization. A close second in fit is the LML-Step, which is also best at $\kappa = 55$, according to the AIC, but at $\kappa = 44$ according to the BIC. In third position we find LML-Poly, and in this case both AIC and BIC converge in indicating $\kappa = 55$ as the model with best fit. So, despite in terms of estimation error the various z functions appeared to perform similarly, they do not do so in terms of information criteria.

For the sake of space we only report and discuss the multimodal aspect of the results. Table 4 reports the number of estimated modes of $mWTP$ distributions. Obviously, MXL (biased and apparently inconsistent) could only retrieve unimodal distributions in all random coefficients. LML models with $\kappa = 22$ and $\kappa = 33$, instead, retrieved bimodal distributions for most of the coefficients. In particular, LML-Poly with $\kappa = 22$ retrieved bimodal distributions for seven coefficients and with $\kappa = 33$ did so for eight $mWTP$ distributions. LML-Step $\kappa = 22$ retrieved bimodal distributions for seven parameters and LML-Step with $\kappa = 33$ for nine $mWTP$ distributions. Similar number of bimodal distributions for random taste coefficients are found in the estimates from LML-spline. Altogether the LML provides a very different characterization of taste distributions, where multimodality and asymmetry emerge as common features.

The histograms reported in the first and second rows of figure 4 are a good illustration of the effect of increasing κ on the number of modes retrieved for the random $mWTP$ for *Taste Weekly* and *Odor Never*: with $\kappa = 22$ the $mWTP$ for the two attribute levels appear to have unimodal distributions, with $\kappa = 44$ they appear bimodal.

With respect to the size of κ, we note that distributions with three modal values were retrieved only by LML models with $\kappa = 44$ and $\kappa = 55$ (e.g. see the bottom histograms in figure 4 for mild and extreme turbidity). In particular, all the specifications with such number of parameters retrieved tri-modal distributions for chlorine odor once per month, chlorine taste once per month, medium and extra degrees of turbidity. All this information would be lost in MXL specifications, and possibly in most other conventional parametric distributions. We note that some multimodality can be captured in means of individual-specific distributions, but those statistics are of difficult interpretation at the population level (see chapter 11 in Train 2009, for a discussion).

Discussion and conclusions

This paper provides results from a large-scale Monte Carlo experiment and an empirical application conducted to investigate the finite sample performance of the recently proposed Logit Mixed Logit (LML) model. We focused on retrieving the underlying heterogeneity distributions of random marginal willingness to pay estimates, with a focus on asymmetric and multimodal data generating processes.

The context is framed around the standard operating conditions for practitioners in agriculture, food and environmental economics. This means that we used a range of sample sizes, experimental designs and panel lengths which are of common use in the published literature on choice analysis for nonmarket valuation, and hence we also focussed on WTP-space utility specifications.

Semiparametric estimators, such as LML, have smaller bias and larger sampling variance at low sample sizes than their more common parametric MXL counterparts, and we measure both. Our result show that the sample size at which bias-efficiency tradeoffs move in LML favour vary depending on the variance around modal values, but at sizes around 500 respondents the overall mean squared error are either comparable to those of MXL or lower. Obviously, these sample sizes need upward adjustments in the presence of more attributes with random coefficients.

Another objective is to identify the optimal number of parameters κ to be adopted in LML model specification for the grid points of probability weights. Our hypothesis, based on previous findings of studies on flexible choice models (Fosgerau and Hess 2007), was that increasing such number increases flexibility and yields better approximations to the true distributions. However, a Monte Carlo study complementary to ours, but with fewer synthetic samples and different focus on DGPs (Bansal, Daziano, and Achtnicht 2018a), drew different conclusions based on information criteria. Using the estimation error as a criterion, our MC results align with those by Fosgerau and Hess (2007), but using as model selection criteria the AIC and BIC our empirical results align with those by Bansal, Daziano, and Achtnicht (2018a). Nevertheless, in the empirical results, trimodal distributions are captured only by LML with high number of parameters κ.

The bias-efficiency tradeoff for LML versus MXL suggests that prior knowledge of the taste distributions with regards to the variance around modal values (i.e. its degree of asymmetry), and to the number of modes one expects, is useful. Specifically, it may inform practitioners on the type of estimator to use: if such variance is small and modal values are few, then the LML can be effective at relatively smaller sample sizes than otherwise.

Importantly, the conditional distributions of *MRAE* show a clear bias versus efficiency tradeoff on the use of efficient experimental design, confirming the caution one must exercise in adopting this design criteria in mis-specified contexts. They also confirm the necessity of relatively large sample sizes and that—at least in our case—the minimum bias one can expect is around 7%, with a maximum of 20% and a median of 12%. Finally, we find that positive relative errors benefit more than negative relative errors from increases in sample sizes and possibly also from other bias-reduction measures. However, this result may be a construct of our choice of DGP, and hence it is not easily generalised.

In our empirical tap water preference study, the LML results suggest a pattern of asymmetric bimodality for tap water quality attributes, such as *taste weekly* and *odor never*. They also show asymmetric trimodality for *mild turbitidty* and *extreme turbidity*. Both features would be missed by the MNL with normal or other unimodal parametric distributions. Addressing such patterns with latent class models would appear not completely satisfactory as these ignore variance around modal values and impose perfect correlation of random coefficients within classes—a restriction that the LML does not impose and for which we find no empirical evidence in our data.

Regulators intending to achieve economically and politically efficient outcomes should be made aware of the multimodal nature of preference for tap water in the tannery district of the Province of Vicenza. The tariff thresholds necessary to trigger majority voting in support of infrastructure investments that deliver only monthly chlorine smell in water and mild turbidity might be lower than those suggested by model estimates obtained with MXL models. This is valuable information to politicians.

Overall, the results of our study do not support the blind use of very flexible mixing distributions: at small sample sizes LML models with a large number of parameters performed worse compared to both LML specifications with low number of parameters and MXL models. Thus, as a general guideline, we suggest to adopt the LML estimator only when a sufficiently large number of observations is available, or when the variance around the possible modes is low. In the presence of poor priors for efficient design and when a 10% bias in marginal WTP estimates is deemed an acceptable cost, our results suggest the use of random or full factorial experimental designs so as to capture the available efficiency gains while expecting some bias. Interestingly, the use of efficient designs based on MNL assumptions and adeaute priors are associated with significantly lower errors also in LML estimates.

While this study provides some insight about LML performance, additional simulation experiments are needed to evaluate the fine-tuning and validate the robustness of our conclusions. For example, we ignored covariance across random coefficients. Future experiments can address that and extend the number of alternatives, of choice situations in the sequence, and of explanatory variables in the utility equation.

Importantly, on the practical side, when asymmetry and multimodality of preference are suspected, analysts can no longer be excused to automatically default on parametric specifications without providing robust theoretical justifications corroborated by empirical evidence. The LML approach is sufficiently practical, general purpose software has been made available for all to use (Train 2016) and it has been recently extended to allow fixed parameters in the specification (Bansal, Daziano, and Achtnicht 2018b).

Notes

[1]They used 6 DGPs, each generating 100 synthetic datasets, which in turn were estimated using 16 specifications, for a total of 9,600 estimates.

[2]For the reader interested in faster estimation algorithms for these category of semi-parametric choice model we refer to Bansal, Daziano, and Guerra (2018).

References

Bansal, P., R.A. Daziano, and M. Achtnicht. 2018a. "Comparison of parametric and semi-parametric representations of unobserved preference heterogeneity in logit models." *Journal of Choice Modelling* 27:97 – 113.

—. 2018b. "Extending the logit-mixed logit model for a combination of random and fixed parameters." *Journal of Choice Modelling* 27:88 – 96.

Bansal, P., R.A. Daziano, and E. Guerra. 2018. "Minorization-Maximization (MM) algorithms for semiparametric logit models: Bottlenecks, extensions, and comparisons." *Transportation Research Part B: Methodological* 115:17–40.

Bazzani, C., M.A. Palma, and R.M. Nayga. 2018. "On the use of flexible mixing distributions in WTP space: an induced value choice experiment." *Australian Journal of Agricultural and Resource Economics* 62:185–198.

Belleflamme, P., W.M.W. Lam, and W. Vergote. 2017. "Price Discrimination and Dispersion under Asymmetric Profiling of Consumers." AMSE Working Papers No. 13, Aix-Marseille School of Economics.

Caputo, V., R. Scarpa, R.M.N. Jr., and D.L. Ortega. 2018. "Are Preferences for Food Quality Attributes Really Normally Distributed? An Analysis using Flexible Mixing Distributions." *Journal of Choice Modelling* 28:10–27.

Chen, Y., and G. Iyer. 2002. "Consumer Addressability and Customized Pricing." *Marketing Science* 21:197–208.

ChoiceMetrics. 2009. "Ngene 1.0 User Manual and Reference Guide."

Daly, A., S. Hess, and K. Train. 2012. "Assuring finite moments for willingness to pay in random coefficients models." *Transportation* 39:19–31.

Ferrini, S., and R. Scarpa. 2007. "Designs with a-priori information for nonmarket valuation with choice-experiments: a Monte Carlo study." *Journal of Environmental Economics and Management* 53(3):342–363.

Fosgerau, M., and S. Hess. 2007. "A comparison of methods for representing random taste heterogeneity in discrete choice models." *European Transport* 42:1–25.

Gouríeroux, C., and A. Monfont. 1996. *Simulation-Based Econometric Methods*. Oxford University Press.

Hensher, D., N. Shore, and K. Train. 2005. "Households' Willingness to Pay for Water Service Attributes." *Environmental and Resource economics* 32:509–531.

Mitchell, R., and R. Carson. 1989. *Using surveys to value public goods : the contingent valuation method*. Washington, D.C.: Resources for the Future.

Papke, L.E., and J.M. Wooldridge. 1996. "Econometric Methods for Fractional Response Variables With an Application to 401 (K) Plan Participation Rates." *Journal of Applied Econometrics* 11:619–632.

Revelt, D., and K. Train. 1998. "Mixed Logit with Repeated Choices: Households' Choices of Appliance Efficiency Level." *Review of Economics and Statistics* 80:647–657.

Rose, J.M., and M.C.J. Bliemer. 2009. "Constructing Efficient Stated Choice Experimental Designs." *Transport Reviews* 29:587–617.

Rungie, C., R. Scarpa, and M. Thiene. 2014. "The influence of individuals in forming collective household preferences for water quality." *Journal of Environmental Economics and Management* 68:161–174.

Sándor, Z., and M. Wedel. 2001. "Designing Conjoint Choice Experiments Using Managers' Prior Beliefs." *Journal of Marketing Research* 38:430–444.

Scarpa, R., C. Franceschinis, and M. Thiene. 2017. "A Monte Carlo Evaluation of the Logit-Mixed Logit under Asymmetry and Multimodality." Working Papers in Economics No. 17/23, University of Waikato.

Scarpa, R., M. Thiene, and F. Marangon. 2008. "Using Flexible Taste Distributions to Value Collective Reputation for Environmentally Friendly Production Methods." *Canadian Journal of Agricultural Economics* 56:145–162.

Scarpa, R., K.G. Willis, and M. Acutt. 2007. "Valuing Externalities from Water Supply: Status Quo, Choice Complexity and Individual Random Effects in Panel Kernel Logit Analysis of Choice Experiments." *Journal of Environmental Planning and Management* 50:449–466.

Thiene, M., R. Scarpa, and J. Louviere. 2015. "Addressing Preference Heterogeneity, Multiple Scales and Attribute Attendance with a Correlated Finite Mixing Model of Tap Water Choice." *Environmental and Resource Economics* 62:637–656.

Train, K. 2009. *Discrete Choice Methods with Simulation*, 2nd ed. New York: Cambridge University Press.

—. 2016. "Mixed Logit with a Flexible Mixing Distribution." *Journal of Choice Modelling* 19:40–53.

Train, K., and M. Weeks. 2005. "Discrete Choice Models in Preference Space and Willing-to-pay Space." In R. Scarpa and A. Alberini, eds. *Applications of simulation methods in environmental and resource economics*. Dordrecht, The Netherlands: Springer Publisher, chap. 1, pp. 1–16.

UNIC. 2010. "Dati strutturali industria conciaria veneta." Working paper, UNCI Unione Nazionale Industria Conciaria (National Tannery Industry Association).

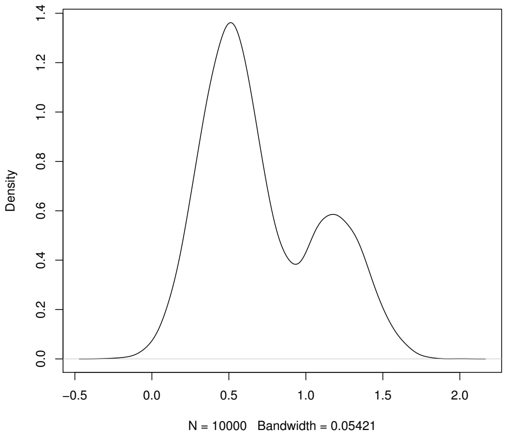
Williams, R. 2009. "Using Heterogeneous Choice Models to Compare Logit and Probit Coefficients across Groups." *Sociological Methods & Research* 37:531–559.

Willis, K., and R. Scarpa. 2002. “Yorkshire Water Customers’ Utility and Willingness-To-Pay For Changes In Water Service Factor Levels: A Stated Choice Analysis.” Unpublished, Report to Yorkshire Waters.

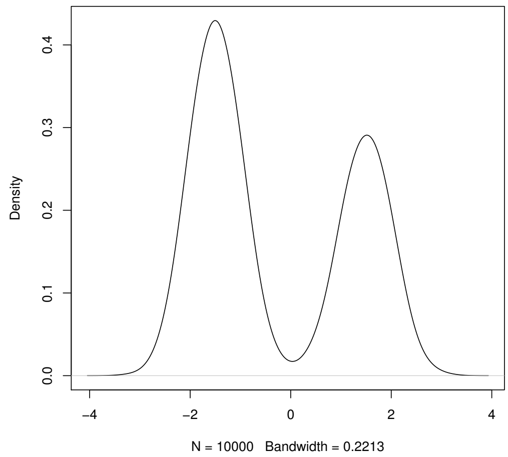
Willis, K., R. Scarpa, and M. Acutt. 2005. “Assessing Water Company Customer Preferences and Willingness-To-Pay for Service Improvements: a stated choice analysis.” *Water Resources Research* 41:W02019.

Wooldridge, J. 2011. “Fractional response models with endogenous explanatory variables and heterogeneity.” Presented at the Stata Conference, Chicago, July 2011.

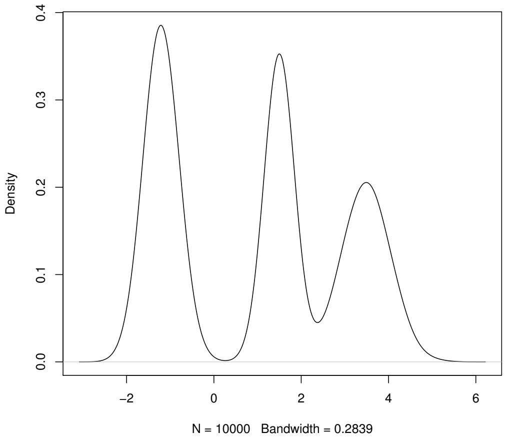
Figures



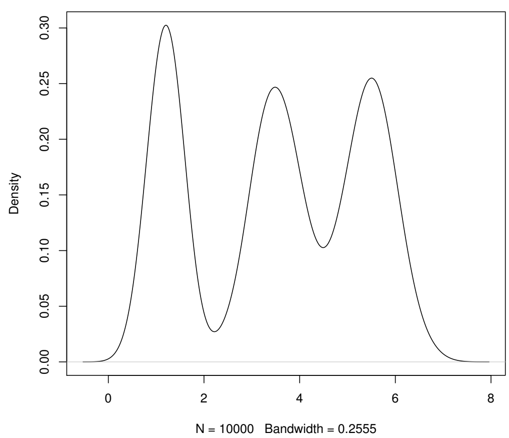
ω^1 in DGP 1 (bimodal)



ω^2 in DGP 1 (bimodal)

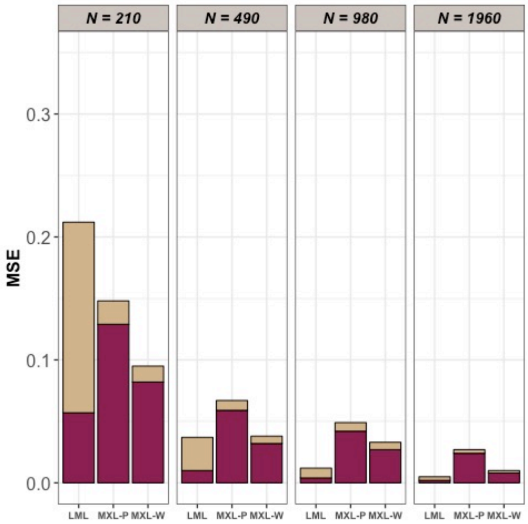


ω^1 in DGP 2 (trimodal)

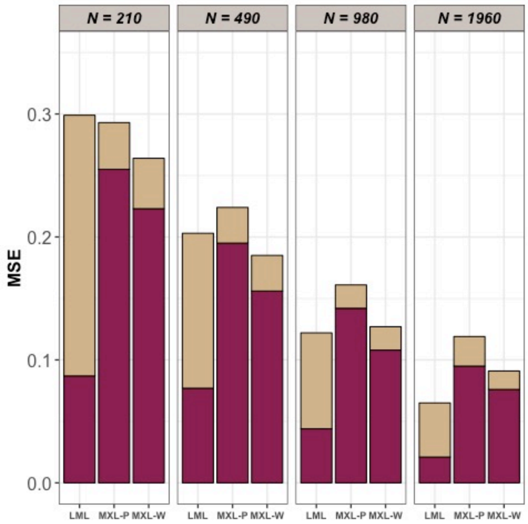


ω^2 in DGP 2 (trimodal)

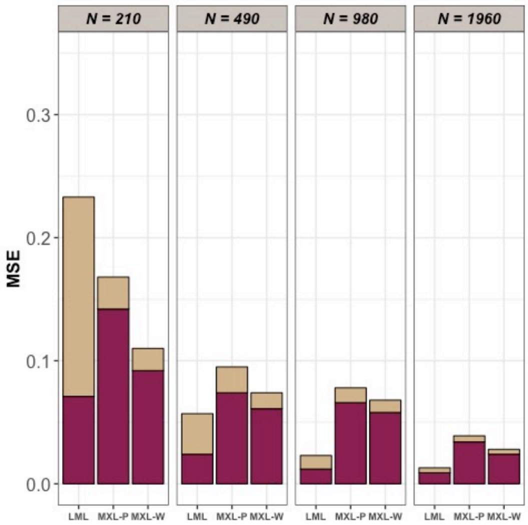
Figure 1. Kernel smoothing plots of $mWTP$ in the 2 DGPs.



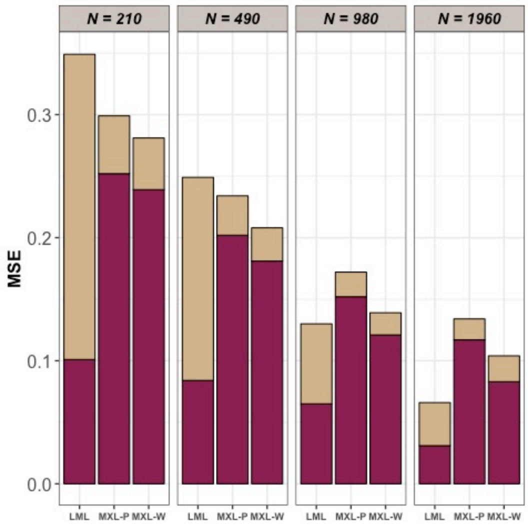
$\hat{\omega}^1$ in DGP 1 (bimodal)



$\hat{\omega}^2$ in DGP 1 (bimodal)



$\hat{\omega}^1$ in DGP 2 (trimodal)



$\hat{\omega}^2$ in DGP 2 (trimodal)

Figure 2. Bias (dark) *vs* variance (light) tradeoff from MSE decomposition

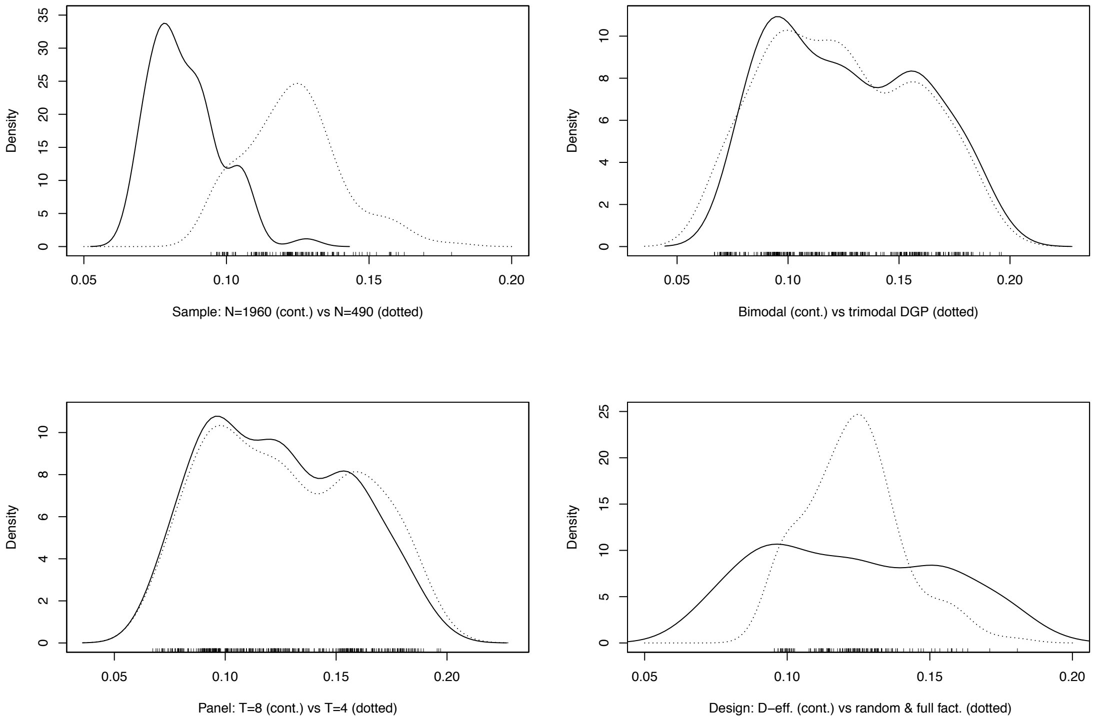
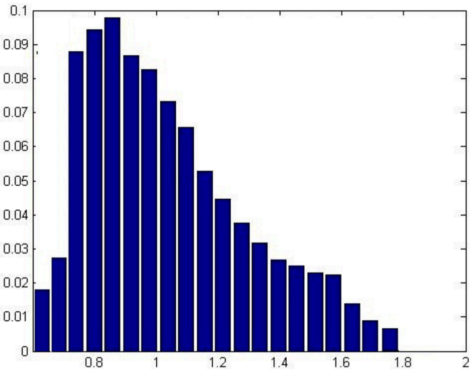
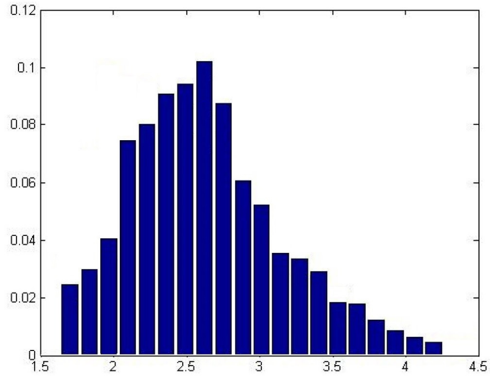


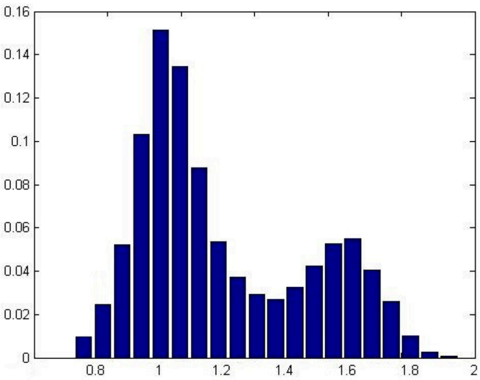
Figure 3. Kernel smoothing of *MRAE* estimates from MC experiment.



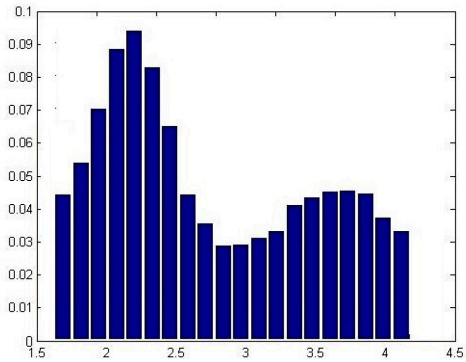
Taste weekly, LML-Spline $\kappa = 22$



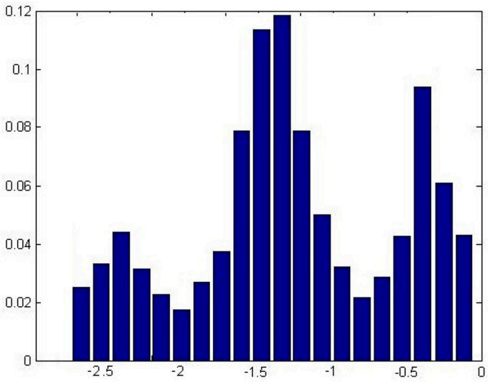
Odor never, LML-Spline $\kappa = 22$



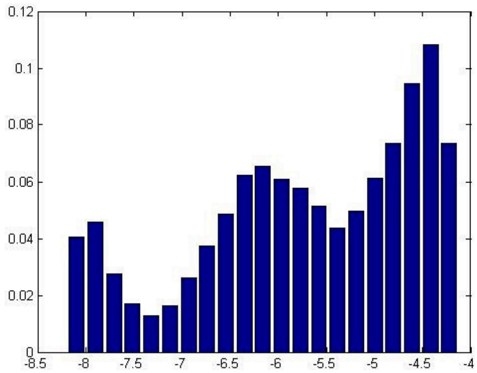
Taste weekly, LML-Spline $\kappa = 44$



Odor never, LML-Spline $\kappa = 44$



Mild turbidity, LML-Spline $\kappa = 44$



Extreme turbidity, LML-Spline $\kappa = 44$

Figure 4. Distributions of WTP value estimates from various LML models.

Tables

Table 1. Surface response model estimates for $\hat{\omega}^1$

	OLS on $MSE = y_r^*$		FRLGT $MRAE = \pi_r$			
Variable	Coeff.	$\|t\|$	Coeff.	$\|t\|$	$\frac{d\pi_r}{ds_g} \times 100$	$\|t\|$
Constant	0.3290	19.22	−1.2657	75.99	—	—
$\sqrt{N}$	−0.0320	3.29	−0.0211	51.77	−0.2292	48.70
dimension of κ	−0.0190	2.61	−0.0056	12.42	−0.0607	12.31
$z(\cdot)$ is polynomial	−0.0015	0.66	0.0093	1.11	0.1013	0.11
$z(\cdot)$ is spline	0.0071	1.56	0.0189	1.45	0.2055	1.45
D-efficient design	−0.0220	2.71	−0.0395	3.21	−0.4306	3.20
choice tasks $T = 8$	−0.0160	3.99	−0.0301	2.66	−0.3271	2.66
DGP is bimodal	−0.0083	3.57	−0.0213	1.98	−0.2315	1.98
$\sqrt{N}\times$ dimension of κ	−0.0072	4.12	—	—	—	—
$\sqrt{N}\times$ D-efficient design	0.0013	0.81	—	—	—	—
$\sqrt{N}\times$ choice tasks	0.0110	0.88	—	—	—	—
$\sqrt{N}\times$ DGP is bimodal	0.0087	6.61	—	—	—	—
dim. of $\kappa \times D$-efficient des.	0.0017	1.33	—	—	—	—
dim. of $\kappa \times$ choice tasks $T = 8$	−0.0005	0.48	—	—	—	—
dim. of $\kappa \times$ DGP is bimodal	0.0019	2.52	—	—	—	—
D-efficient des. $\times$ choice tasks $T = 8$	0.0042	1.50	—	—	—	—
D-efficient des. $\times$ DGP is bimodal	0.0028	0.66	—	—	—	—
choice tasks $T = 8\times$ DGP is bimodal	0.0435	0.79	—	—	—	—
$N = 720$	$\bar{R}^2 = 0.89$	F-stat. = 87.45	Pseudo-$\mathscr{L}^* - 268.91$		Wald $\chi^2 = 7,595.54$	

Table 2. Surface response model estimates for $MARE$ for $\hat{\omega}^1$

	OLS								FRLGT			
	Pooled sample				*MNRE* sample		*MPRE* sample		*MNRE* sample		*MPRE* sample	
	$\frac{d\pi_r}{ds_g} \times 100$	$\lvert t \rvert$	$\frac{d\pi_r}{ds_g} \times 100$	$\lvert t \rvert$	$\frac{d\pi_r^-}{ds_g} \times 100$	$\lvert t \rvert$	$\frac{d\pi_r^+}{ds_g} \times 100$	$\lvert t \rvert$	$\frac{d\pi_r^-}{ds_g} \times 100$	$\lvert t \rvert$	$\frac{d\pi_r^+}{ds_g} \times 100$	$\lvert t \rvert$
Constant	19.54	73.40	18.26	48.13	16.93	49.66	19.59	48.89				
$\sqrt{N}$	−0.1873	24.31	−0.1715	14.52	−0.1598	16.79	−0.1831	16.38	−0.1383	15.93	−0.1585	15.57
Dimension of κ	−0.0258	3.10	−0.0258	2.25	−0.0250	2.34	−0.0265	2.13	−0.0048	0.51	−0.0037	0.33
$z(\cdot)$ = Polynomial	0.0915	1.49	0.0915	1.46	0.0950	1.13	−0.0868	0.92	0.0953	1.15	0.0871	0.93
$z(\cdot)$ = Spline	0.1945	1.75	0.1945	1.76	0.1827	1.44	−0.2063	1.40	0.1826	1.45	0.2062	1.41
D-efficient	−0.4076	4.20	−0.4076	4.24	−0.3864	3.42	−0.4301	3.38	−0.3884	3.56	−0.4322	3.42
choice tasks ($T = 8$)	−0.3063	2.59	−0.3063	3.36	−0.2871	2.72	−0.3256	2.49	−0.2877	2.72	−0.3262	2.80
Bimodal	−0.2467	3.55	−0.2467	2.65	−0.2287	2.43	−0.2658	2.41	−0.2170	2.16	−0.2522	2.15
Dimension of $\kappa \times \sqrt{N}$	−0.0013	5.37	−0.0013	3.70	−0.0012	3.87	−0.0015	4.10	−0.0023	7.84	−0.0027	8.01
Dummy for *MNRE*	−2.5664	17.71										
Dummy for *MNRE* $\times \sqrt{N}$	−0.0031	7.18										
R^2	0.797		0.719		0.781		0.781		Pseudo-$\mathscr{L}^*$	−249.30	−274.61	
$\bar{R}^2$	0.795		0.718		0.779		0.779					
N	1,440				720							

Note: ***MNRE*** **mean of** ***negative*** **relative errors,** ***MPRE*** **mean of** ***positive*** **relative errors.**

Table 3. Information criteria for tap water models.

Model	κ	$\ln \mathscr{L}^*$	**AIC**	**BIC**
MXL Pref.	11	−2,932	5,821	5,823
MXL WTP	11	−2,908	5,794	5,771
LC 2 classes	23	−2,896	5,741	5,753
LC 3 classes	35	−2,877	5,724	5,745
LML-Poly	22	−2,818	5,614	5,637
LML-Poly	33	−2,774	5,526	5,549
LML-Poly	44	−2,732	5,442	5,465
LML-Poly	55	−2,718	5,414	5,437
LML-Step	22	−2,802	5,582	5,605
LML-Step	33	−2,758	5,494	5,517
LML-Step	44	−2,716	5,410	5,503
LML-Step	55	−2,702	5,382	5,505
LML-Spline	22	−2,786	5,550	5,573
LML-Spline	33	−2,742	5,462	5,485
LML-Spline	44	−2,700	5,378	**5,401**
LML-Spline	55	−2,686	**5,350**	5,412

Table 4. Modal values of distributions of attributes' coefficients (Empirical application)

		Odor			Taste			Turbidity			Stain
Model/Attribute	κ	**Weekly**	**Monthly**	**Never**	**Weekly**	**Monthly**	**Never**	**Mild**	**Medium**	**Extra**	**Present**
MXL Pref.	11	1	1	1	1	1	1	1	1	1	1
MXL WTP	11	1	1	1	1	1	1	1	1	1	1
LML-Poly	22	2	2	2	1	2	1	1	2	1	2
LML-Poly	33	1	2	2	1	2	2	2	2	1	2
LML-Poly	44	2	3	2	2	3	2	3	2	3	2
LML-Poly	55	2	3	2	2	3	2	3	2	3	2
LML-Step	22	1	2	1	1	2	2	2	2	1	2
LML-Step	33	2	2	2	2	2	1	1	2	1	2
LML-Step	44	2	3	3	2	2	2	3	2	3	2
LML-Step	55	2	2	3	2	3	2	3	2	3	2
LML-Spline	22	2	2	1	1	2	1	1	2	1	2
LML-Spline	33	2	2	2	2	2	1	2	2	2	2
LML-Spline	44	2	3	2	2	2	2	3	2	3	2
LML-Spline	55	2	3	2	2	2	2	3	3	3	2

Appendix to:
Logit mixed logit under asymmetry and multimodality of WTP: a Monte Carlo evaluation

Results from the Monte Carlo experiment

In what follows we describe the results with focus on those obtained from datasets generated with the *D*-error minimizing design. Similar results were also obtained with regards to the accuracy measures *MSE* and *MRAE* and for this reason we limit our appendix to the *MSE* values across different models and DGPs. All omitted results are available from the authors upon request.

Model fit

Table A3 reports the information criteria for models estimated on datasets with bimodal DGP and D-efficient experimental design with four choice tasks per simulated respodent ($T = 4$). Model fit statistics suggest that the increase in the number of parameters in all LML variants improves the loglikelihood value at convergence at every sample size, but it does not necessarily improve the model fit in terms of AIC and BIC. At small sample sizes (from 70 to 490 respondents) MXL-N models outperform LML models in terms of both AIC and BIC. As it concerns the performance of LML variants, at small sample sizes the specifications with $\kappa = 24$are consistently the best performing ones in terms of AIC and BIC. This suggests that at small sample sizes, more flexible LML mixing distributions (which require more parameters) do not necessarily yield a large enough gain in the likelihood values to make these models preferable in terms of AIC and BIC to both LML models with fewer parameters and MXL-N models. At large sample sizes, instead, the more flexible specifications ($\kappa = 36, 48$) seem to fit the data better than the less flexible ones. At $N = 980$ the LML-Spline with $\kappa = 48$ is the best performing LML variant in terms of AIC, and the LML-Step with $\kappa = 36$ parameters according to BIC. At the largest sample size ($N = 1980$), the LML-Spline with 48 parameters is the best performing model according to both AIC and BIC. It is also interesting to note that at the largest sample size all LML variants outperform the MXL-N models according to all the information criteria. Similar results were retrieved for model estimated from dataset generated with trimodal DGP (Table A4) and for datasets with eight choice tasks per simulated respondent, which we omit for the sake of brevity.

Mean squared errors

Short panel results, $T = 4$, bimodal distributions

Table A5 reports MSE values for estimates of the mean $mWTP$ for attribute 1 and attribute 2 as retrieved from datasets with four choice tasks per respondent, with DGP 1 that implemented asymmetric bimodal distributions of the real parameters. It is immediately noticeable that, given ω, the value of MSE decreases as N increases: accuracy is increased by larger samples. For small samples (simulated respondents $N = 70$ and $N = 210$) the best performing model—that is, the one with the lowest MSE (and $MRAE$)—is the MXL-N WTP space, which outperforms all LML models. A bias variance tradeoff seems to take place at this level. At intermediate sample sizes ($N = 490$ and $N = 980$ simulated respondents) some of the LML specifications outperformed the MXL in WTP-space (e.g. LML-poly with $\kappa = 36$ for ω^1 and LML-Spline with $\kappa = 48$ and $\kappa = 24$ for ω_2), but it is only at large sample sizes ($N = 1,960$) that LML models consistently outperformed the MNL-N WTP for some value of κ. At $N = 1,960$ there is also a clear improvement in performance by LML models with higher dimensions of κ. Among LML models based on step functions and splines the best model specifications were those with $\kappa = 48$, whereas the best model specification among LML-Poly models was the one with $\kappa = 36$ according to both *MSE* and *MRAE*.

In terms of identification of the optimal number of parameters κ to be adopted in LML models for both bimodal coefficients, we obtain no clear indication at such sample sizes. According to the *MSE* values for ω^1, for the LML-Poly the best specification is the one with $\kappa = 24$, followed by $\kappa = 48$ and then $\kappa = 36$. Moving to the results for LML-Step, the best performing models are those with high number of κ (36 and 48). Finally, among LML-Spline, the best performing model specification is the one with $\kappa = 24$, followed by the one with $\kappa = 36$. For to the *MSE* for the second coefficient ω_2, the best performing LML-Poly has $\kappa = 24$ and 48; for the LML-Step $\kappa = 48$, while for the LML-Spline is the one with $\kappa = 36$.

The second important distribution feature is its spread, often measured by the standard deviation. The *MSE* for these statistics of the Monte Carlo results are reported in Table A6. As for the means, at the smallest sample size the MXL-N WTP outperforms all models (and it always outperforms the MXL-N in preference space), but already at $N = 210$ we have LML-Step with $\kappa = 36$ that does better and at higher sample sizes LML models do better both more frequently and more consistently, especially at high values of κ.

Long panel results, $T = 8$, bimodal distributions

Tables A7 and A8 reports the same statistics as above, but for the longer panel with eight choice tasks per respondent ($T = 8$). So, the number of choices are doubled at each sample size. Doubling the number of responses collected from each respondent obviously sharpens the estimation of the distributions of taste, as it allows for both better panel designs and more information from more numerous choices. Whether and at what sample size this difference becomes apparent with respect to $T = 4$ is an empirical question we try to answer here. The results from datasets with small sample size ($N = 70$ and $N = 210$) are similar to those retrieved from datasets with four choice scenarios per respondent, in that the MXL model outperforms the LML specifications and there are no clear indications about the effect of increasing the number of parameters of LML specifications.

However, *MSE* for both means and standard deviations show that flexible LML specifications consistently surpass the MXL model at both intermediate and large sample sizes. Similarly to the short panel results, for $N = 980$ respondents, each LML specification outperformed the MXL model for some value of a κ. This suggests that increasing the number of observations per respondent (a longer panel) does not seem to allow analysts to retrieve substantively more accurate estimates with LML models at smaller sample sizes. At both $N = 980$ and $N = 1,960$, it is also apparent that model specifications with large κ outperform the others.

Short panel results, $T = 4$, trimodal distributions

We now move to the results for the choice data generated under the DGP 2 with asymmetric trimodal distributions for ω_n^1 and ω_n^2 reported in Tables A9 and A10 for the case with short panel. Results are similar to those retrieved for the first set of coefficients in that the MXL-N WTP model always outperforms the MNL-N Pref. and does so for LML models at small sample sizes. The main difference is that in this case, already a $N = 490$, so at intermediate sample sizes, the *MSE* for LML are frequently smaller than those for the MXL-N WTP. It seems to be the case that with a trimodal distribution DGP flexible distribution models are more accurate than MXL-N at lower sample sizes, even with short panel, especially the LML-Step and LML-Spline.

Long panel results, $T = 8$, trimodal distributions

Tables A13 and A14 report the same statistics for the long panel. No noticeable difference is found from the results obtained for the short panel, indicating that doubling the number of choices per respondent does not substantially change the tradeoff between bias and sample size.

Bimodality

Tables A13-A16 report the means and standard deviations of modal estimates of distributions of ω_n^1 and ω_n^2 from the various model specifications in both the short panels and long panels. They all have in common the bimodal DGP 1 as true process.

The first important observation concerns the number of modal values retrieved from different model specification. Naturally, MXL-N models (both in preference and WTP space) are inherently unimodal and cannot, by their very nature, imply bimodal distributions, but they are expected to retrieve a mean/mode/median at an intermediate value between the modes of the underlying DGPs. Indeed the results confirm this. Instead, LML models can retrieve bimodal distributions and do so in our experiment, with a degree of accuracy that increases with the sample size. This confirms that LML models are able to approximate better the shape of the true underlying distributions of random coefficients, and should always be considered when unimodality is not well supported a-priori, as it is often the case.

The second objective of the analysis was to identify how close the local maxima and minima retrieved from different LML specification were to the true ones. In this sense, it appears that increasing both the sample size and κ increases the accuracy of the estimates. In fact, the values that are closer to the real ones were obtained from LML specifications with $\kappa = 48$ estimated using datasets with $N = 1,980$. Of course, one can also compute

MSE and *MRAE* values for modal estimates and compare them across LML models. We have those results, but chose not to discuss them here.

Trimodality

Tables A18-A20 report the number of modal values from model estimated on data from DGP 2 (trimodal real distributions of mWTPs). As for the bimodal case, MXL model cannot retrieve the complex form of the real distributions, and deliver unimodal distributions at intermediate values of the modes in the real data. LML specifications with $\kappa = 12$, instead, always retrieve distributions with two modal values, instead of three. On the other hand, LML specifications with $\kappa = 48$ always correctly retrieve distribution with three modal values. Finally, LML specifications with intermediate $\kappa = 24 - 36$ retrieve distributions with three modal values at intermediate and large sample sizes, but bimodal distributions at lower N. As in previous cases, it is apparent that increasing sample sizes and κ increases the accuracy of estimates. In fact, modal values of distributions retrieved from model estimated from large datasets are closer to the DGP values.

Overall the results suggest that LML models may outperform the standard MXL-N specifications and represent more accurately complex distributions, but do so especially at large N. With regards to the optimal κ to be used in LML models, it seems that high κ values should be considered, but unsurprisingly they work better at large N.

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Figures

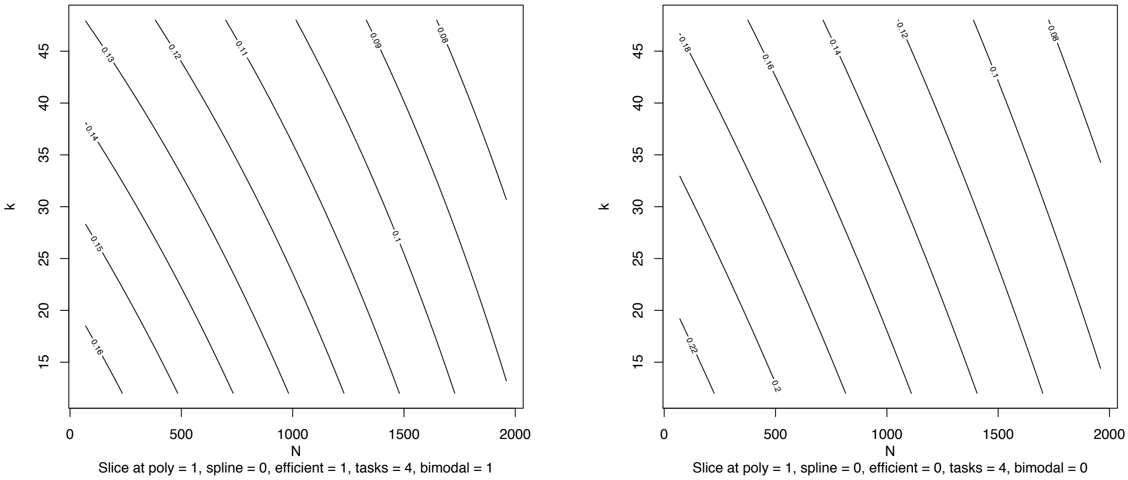

Figure A1. Contour plots.

Tables

Table A2. Descriptive statistics for variables used in surface response models

Dependent variables	**Mean**	**Median**	**Max**	**Min**
MSE	0.188	0.184	0.361	0.004
MRAE	0.123	0.125	0.197	0.068
MNRE	0.111	0.109	0.176	0.061
MPRE	0.129	0.127	0.203	0.069
Continuous indipendent variables	**Mean**	**Median**	**Max**	**Min**
N	742	490	1960	70
Dimensions of κ	30	30	12	48
Binary indipendent variables	**Frequency of value = 1**			
$z(\cdot)$ is Polynomial	0.333			
$z(\cdot)$ is Spline	0.333			
D-efficient design	0.333			
Choice tasks T = 8	0.500			
DGP is bimodal	0.500			

Table A3. Information criteria (bimodal, $T = 4$)

Model		$N = 70$			$N = 210$			$N = 490$			$N = 980$			$N = 1,960$		
Model	κ	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**
MXL-N Pref.	6	-1003.2	2018.4	2031.9	-1775.7	3563.3	3583.4	-2548.1	5108.3	5133.4	-3665.6	7343.2	7372.5	-5582.4	11176.9	11210.4
MXL-N WTP	6	-1005.6	2023.2	2036.7	-1779.9	3571.8	3591.9	-2564.3	5140.6	5165.7	-3677.1	7366.2	7395.5	-5565.4	11142.8	11176.4
LML-Poly	12	-1033.7	2091.4	2118.4	-1820.7	3665.4	3705.6	-2569.7	5163.4	5213.8	-3635.9	7295.8	7354.4	-5394.9	10813.7	10880.8
	24	-1006.2	2060.3	2114.3	-1785.9	3619.8	3700.1	-2530.6	5109.2	5209.9	-3562.2	7172.4	7289.7	-5351.7	10751.4	10885.6
	36	-1001.8	2075.5	2156.4	-1778.1	3628.2	3748.7	-2519.5	5110.9	5261.9	-3541.5	7154.9	7330.9	-5318.2	10708.3	10909.6
	48	-1000.3	2096.5	2204.4	-1775.4	3646.9	3807.5	-2515.6	5127.3	5328.6	-3517.0	7130.0	7364.6	-5283.0	10662.0	10930.4
LML-Step	12	-1029.4	2082.8	2109.8	-1816.6	3657.3	3697.4	-2591.9	5207.8	5258.1	-3627.7	7279.4	7338.0	-5394.3	10812.5	10879.6
	24	-1000.7	2049.3	2103.3	-1781.2	3610.3	3690.6	-2571.7	5191.3	5292.0	-3566.5	7181.0	7298.3	-5353.1	10754.2	10888.4
	36	-1000.3	2072.6	2153.5	-1775.5	3623.1	3743.6	-2520.8	5113.5	5264.5	-3514.2	7100.4	7276.4	-5306.0	10684.0	10885.3
	48	-998.0	2091.9	2199.8	-1771.4	3638.7	3799.4	-2509.8	5115.6	5316.9	-3506.8	7109.6	7344.2	-5281.0	10658.0	10926.4
LML-Spline	12	-1022.8	2069.6	2096.6	-1820.8	3665.6	3705.7	-2572.6	5169.3	5219.6	-3643.7	7311.4	7370.1	-5401.5	10827.0	10894.1
	24	-1010.8	2069.5	2123.5	-1789.0	3626.1	3706.4	-2527.3	5102.6	5203.3	-3582.4	7212.8	7330.1	-5363.2	10774.5	10908.7
	36	-1007.2	2086.3	2167.2	-1782.7	3637.5	3757.9	-2513.3	5098.6	5249.6	-3538.2	7148.4	7324.3	-5317.0	10706.1	10907.4
	48	-1005.3	2106.5	2214.4	-1779.3	3654.6	3815.2	-2508.3	5112.7	5314.0	-3489.2	7074.3	7308.9	-5255.8	10607.5	10875.9

Table A4. Information criteria (trimodal, $T = 4$)

		$N = 70$			$N = 210$			$N = 490$			$N = 980$			$N = 1,960$		
Model	κ	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**	$\ln \mathscr{L}^*$	**AIC**	**BIC**
MXL-N Pref.	6	-1092.5	2197.0	2210.5	-1865.2	3742.5	3762.6	-2638.0	5288.0	5313.2	-4022.8	8057.7	8087.0	-6079.1	12170.1	12203.7
MXL-N WTP	6	-1095.1	2202.2	2215.7	-1869.5	3751.0	3771.1	-2654.5	5321.0	5346.1	-4024.9	8061.8	8091.2	-6094.6	12201.2	12234.8
LML-Poly	12	-1132.1	2288.3	2315.3	-1921.3	3866.5	3906.7	-2660.7	5345.3	5395.7	-4096.9	8217.8	8276.5	-5899.8	11823.6	11890.7
	24	-1092.6	2233.2	2287.2	-1875.8	3799.7	3880.0	-2650.6	5349.1	5449.8	-4053.9	8155.7	8273.0	-5855.6	11759.1	11893.3
	36	-1091.2	2254.4	2335.4	-1868.1	3808.1	3928.6	-2649.5	5371.0	5522.0	-4013.0	8097.9	8273.9	-5808.9	11689.9	11891.2
	48	-1089.3	2274.5	2382.5	-1864.5	3824.9	3985.6	-2635.1	5366.1	5567.4	-4001.7	8099.5	8334.1	-5760.1	11616.2	11884.6
LML-Step	12	-1119.9	2263.9	2290.9	-1917.0	3858.0	3898.1	-2654.1	5332.2	5382.6	-4098.3	8220.6	8279.2	-5889.5	11803.1	11870.2
	24	-1090.7	2229.4	2283.4	-1870.3	3788.6	3868.9	-2641.1	5330.2	5430.9	-4062.7	8173.4	8290.7	-5848.4	11744.7	11878.9
	36	-1090.0	2252.1	2333.0	-1865.5	3802.9	3923.4	-2635.6	5343.2	5494.2	-4002.7	8077.5	8253.4	-5797.5	11667.1	11868.3
	48	-1087.4	2270.7	2378.7	-1860.4	3816.8	3977.4	-2629.4	5354.8	5556.1	-3960.5	8017.0	8251.6	-5762.5	11621.0	11889.4
LML-Spline	12	-1124.0	2271.9	2298.9	-1913.7	3851.5	3891.6	-2663.7	5351.4	5401.7	-4103.5	8231.1	8289.7	-5900.6	11825.1	11892.2
	24	-1101.1	2250.2	2304.2	-1879.2	3806.4	3886.7	-2657.4	5362.7	5463.4	-4054.8	8157.6	8274.9	-5849.4	11746.8	11881.0
	36	-1097.4	2266.8	2347.7	-1872.9	3817.7	3938.2	-2653.9	5379.9	5530.9	-4025.7	8123.5	8299.4	-5818.6	11709.2	11910.4
	48	-1095.2	2286.4	2394.4	-1868.8	3833.5	3994.2	-2643.8	5383.6	5584.9	-4005.9	8107.7	8342.3	-5755.0	11606.1	11874.4

Table A5. ***MSE*** **for means of random coefficients in DGP 1 (bimodal, $T = 4$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2
MXL-N Pref.	6	0.276	0.436	0.234	0.383	0.142	0.299	0.108	0.200	0.055	0.135
MXL-N WTP	6	0.165	0.301	0.095	0.281	0.038	0.208	0.013	0.128	0.009	0.104
LML-Poly	12	0.224	0.429	0.132	0.311	0.059	0.294	0.042	0.245	0.014	0.137
	24	0.268	0.404	0.104	0.359	0.064	0.225	0.022	0.125	0.009	0.077
	36	0.361	0.554	0.215	0.379	0.036	0.301	0.019	0.095	0.004	0.059
	48	0.276	0.395	0.212	0.349	0.037	0.229	0.012	0.097	0.005	0.054
LML-Step	12	0.245	0.407	0.237	0.312	0.039	0.231	0.054	0.248	0.023	0.160
	24	0.209	0.405	0.149	0.384	0.072	0.201	0.061	0.132	0.014	0.104
	36	0.212	0.326	0.174	0.304	0.094	0.225	0.026	0.101	0.012	0.056
	48	0.261	0.365	0.141	0.315	0.115	0.271	0.013	0.093	0.007	0.052
LML-Spline	12	0.288	0.485	0.243	0.322	0.069	0.235	0.089	0.219	0.021	0.084
	24	0.197	0.423	0.139	0.332	0.089	0.191	0.008	0.148	0.014	0.053
	36	0.263	0.456	0.201	0.463	0.126	0.231	0.022	0.128	0.006	0.049
	48	0.309	0.445	0.282	0.312	0.044	0.188	0.008	0.105	0.006	0.046

Table A6. ***MSE*** **for st. dev. of random coefficients in DGP 1 (bimodal, $T = 4$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
MXL-N Pref.	6	0.463	0.693	0.420	0.629	0.360	0.516	0.322	0.401	0.283	0.359
MXL-N WTP	6	0.370	0.542	0.319	0.516	0.293	0.475	0.286	0.376	0.246	0.346
LML-Poly	12	0.467	0.639	0.481	0.672	0.361	0.591	0.342	0.487	0.284	0.384
	24	0.484	0.628	0.341	0.551	0.351	0.487	0.265	0.329	0.245	0.297
	36	0.449	0.765	0.419	0.714	0.305	0.614	0.261	0.317	0.220	0.278
	48	0.411	0.645	0.329	0.561	0.267	0.495	0.246	0.293	0.214	0.267
LML-Step	12	0.392	0.597	0.421	0.554	0.341	0.579	0.363	0.469	0.252	0.373
	24	0.401	0.644	0.492	0.638	0.362	0.516	0.295	0.360	0.223	0.346
	36	0.424	0.553	0.425	0.505	0.330	0.440	0.255	0.333	0.222	0.289
	48	0.408	0.556	0.367	0.551	0.299	0.489	0.245	0.313	0.207	0.275
LML-Spline	12	0.421	0.591	0.775	0.649	0.312	0.421	0.305	0.449	0.282	0.378
	24	0.424	0.651	0.324	0.560	0.269	0.413	0.279	0.352	0.241	0.263
	36	0.467	0.718	0.449	0.624	0.334	0.582	0.262	0.342	0.232	0.288
	48	0.488	0.681	0.489	0.665	0.426	0.412	0.253	0.345	0.218	0.248

Table A7. ***MSE*** **for means of random coefficients in DGP 1 (bimodal, $T = 8$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2
MXL-N Pref.	6	0.185	0.277	0.168	0.252	0.144	0.207	0.129	0.161	0.113	0.144
MXL-N WTP	6	0.148	0.217	0.128	0.207	0.117	0.190	0.115	0.151	0.099	0.139
LML-Poly	12	0.187	0.256	0.193	0.269	0.145	0.237	0.137	0.195	0.114	0.154
	24	0.194	0.251	0.137	0.221	0.141	0.195	0.106	0.132	0.098	0.119
	36	0.180	0.306	0.168	0.286	0.122	0.246	0.105	0.127	0.088	0.111
	48	0.165	0.258	0.132	0.225	0.107	0.198	0.099	0.117	0.086	0.107
LML-Step	12	0.157	0.239	0.169	0.222	0.137	0.232	0.145	0.188	0.101	0.149
	24	0.161	0.258	0.197	0.255	0.145	0.207	0.118	0.144	0.085	0.139
	36	0.170	0.221	0.170	0.202	0.132	0.176	0.102	0.133	0.089	0.116
	48	0.163	0.223	0.147	0.221	0.120	0.196	0.098	0.125	0.083	0.110
LML-Spline	12	0.169	0.237	0.210	0.260	0.125	0.169	0.122	0.180	0.113	0.151
	24	0.170	0.261	0.130	0.224	0.108	0.165	0.112	0.141	0.097	0.105
	36	0.187	0.287	0.180	0.250	0.134	0.233	0.105	0.137	0.093	0.115
	48	0.195	0.273	0.196	0.266	0.171	0.165	0.101	0.138	0.087	0.099

Table A8. ***MSE*** **for st. dev. of random coefficients in DGP 1 (bimodal, $T = 8$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
MXL-N Pref.	6	0.204	0.305	0.185	0.277	0.158	0.228	0.142	0.177	0.124	0.158
MXL-N WTP	6	0.163	0.239	0.141	0.228	0.129	0.209	0.127	0.166	0.109	0.153
LML-Poly	12	0.206	0.282	0.212	0.296	0.160	0.261	0.151	0.215	0.125	0.169
	24	0.213	0.276	0.151	0.243	0.155	0.215	0.117	0.145	0.108	0.131
	36	0.198	0.337	0.185	0.315	0.134	0.271	0.116	0.140	0.097	0.122
	48	0.182	0.284	0.145	0.248	0.118	0.218	0.109	0.129	0.095	0.118
LML-Step	12	0.173	0.263	0.186	0.244	0.151	0.255	0.160	0.207	0.111	0.164
	24	0.177	0.284	0.217	0.281	0.160	0.228	0.130	0.158	0.094	0.153
	36	0.187	0.243	0.187	0.222	0.145	0.194	0.112	0.146	0.098	0.128
	48	0.179	0.245	0.162	0.243	0.132	0.216	0.108	0.138	0.091	0.121
LML-Spline	12	0.186	0.261	0.231	0.286	0.138	0.186	0.134	0.198	0.124	0.166
	24	0.187	0.287	0.143	0.246	0.119	0.182	0.123	0.155	0.107	0.116
	36	0.206	0.316	0.198	0.275	0.147	0.256	0.116	0.151	0.102	0.127
	48	0.215	0.300	0.216	0.293	0.188	0.182	0.111	0.152	0.096	0.109

Table A9. ***MSE*** **for means of random coefficients in DGP 2 (trimodal, $T = 4$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2
MXL-N Pref.	6	0.248	0.392	0.211	0.345	0.128	0.269	0.097	0.180	0.050	0.122
MXL-N WTP	6	0.149	0.271	0.086	0.253	0.074	0.187	0.068	0.115	0.008	0.094
LML-Poly	12	0.202	0.386	0.119	0.280	0.053	0.265	0.038	0.221	0.013	0.123
	24	0.241	0.364	0.094	0.323	0.058	0.203	0.020	0.113	0.008	0.069
	36	0.325	0.499	0.194	0.341	0.032	0.271	0.017	0.086	0.004	0.053
	48	0.248	0.356	0.191	0.314	0.033	0.206	0.011	0.087	0.005	0.049
LML-Step	12	0.221	0.366	0.213	0.281	0.035	0.208	0.049	0.223	0.021	0.144
	24	0.188	0.365	0.134	0.346	0.065	0.181	0.035	0.119	0.013	0.094
	36	0.191	0.293	0.157	0.274	0.085	0.203	0.023	0.091	0.011	0.050
	48	0.235	0.329	0.127	0.284	0.104	0.244	0.012	0.084	0.006	0.047
LML-Spline	12	0.259	0.437	0.219	0.290	0.062	0.212	0.080	0.197	0.019	0.076
	24	0.177	0.381	0.125	0.299	0.080	0.172	0.057	0.133	0.013	0.048
	36	0.237	0.410	0.181	0.417	0.113	0.208	0.020	0.115	0.005	0.044
	48	0.278	0.401	0.254	0.281	0.040	0.169	0.007	0.095	0.005	0.041

Table A10. ***MSE*** **for standard deviations of random coefficients in DGP 2 (trimodal, $T = 4$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
MXL-N Pref.	6	0.426	0.633	0.392	0.581	0.341	0.477	0.352	0.368	0.304	0.342
MXL-N WTP	6	0.371	0.500	0.301	0.479	0.281	0.443	0.344	0.398	0.294	0.328
LML-Poly	12	0.432	0.592	0.447	0.614	0.340	0.549	0.312	0.469	0.283	0.356
	24	0.452	0.577	0.319	0.510	0.329	0.447	0.271	0.367	0.262	0.323
	36	0.413	0.701	0.388	0.654	0.288	0.566	0.216	0.369	0.262	0.257
	48	0.383	0.596	0.307	0.522	0.255	0.462	0.230	0.304	0.244	0.250
LML-Step	12	0.368	0.551	0.389	0.511	0.318	0.538	0.365	0.424	0.256	0.306
	24	0.373	0.592	0.457	0.584	0.337	0.482	0.271	0.357	0.233	0.290
	36	0.394	0.507	0.392	0.470	0.315	0.410	0.272	0.374	0.223	0.268
	48	0.382	0.515	0.342	0.506	0.282	0.458	0.272	0.332	0.192	0.262
LML-Spline	12	0.395	0.542	0.707	0.600	0.293	0.391	0.339	0.452	0.289	0.281
	24	0.394	0.597	0.306	0.517	0.252	0.383	0.290	0.399	0.280	0.271
	36	0.432	0.659	0.418	0.576	0.312	0.540	0.315	0.383	0.285	0.262
	48	0.453	0.626	0.456	0.610	0.401	0.385	0.280	0.326	0.247	0.257

Table A11. ***MSE*** **for means of random coefficients in DGP 2 (trimodal, $T = 8$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2	ω^1	ω^2
MXL-N Pref.	6	0.167	0.249	0.151	0.227	0.130	0.186	0.116	0.145	0.102	0.130
MXL-N WTP	6	0.133	0.195	0.115	0.186	0.105	0.171	0.104	0.136	0.089	0.125
LML-Poly	12	0.168	0.230	0.174	0.242	0.131	0.213	0.123	0.176	0.103	0.139
	24	0.175	0.226	0.123	0.199	0.127	0.176	0.095	0.119	0.088	0.107
	36	0.162	0.275	0.151	0.257	0.110	0.221	0.095	0.114	0.079	0.100
	48	0.149	0.232	0.119	0.203	0.096	0.178	0.089	0.105	0.077	0.096
LML-Step	12	0.141	0.215	0.152	0.200	0.123	0.209	0.131	0.169	0.091	0.134
	24	0.145	0.232	0.177	0.230	0.131	0.186	0.106	0.130	0.077	0.125
	36	0.153	0.199	0.153	0.182	0.119	0.158	0.092	0.120	0.080	0.104
	48	0.147	0.201	0.132	0.199	0.108	0.176	0.088	0.113	0.075	0.099
LML-Spline	12	0.152	0.213	0.189	0.234	0.113	0.152	0.110	0.162	0.102	0.136
	24	0.153	0.235	0.117	0.202	0.097	0.149	0.101	0.127	0.087	0.095
	36	0.168	0.258	0.162	0.225	0.121	0.210	0.095	0.123	0.084	0.104
	48	0.176	0.246	0.176	0.239	0.154	0.149	0.091	0.124	0.078	0.089

Table A12. ***MSE*** **for standard deviations of random coefficients in DGP 2 (trimodal, $T = 4$)**

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
MXL-N Pref.	6	0.184	0.275	0.167	0.249	0.142	0.205	0.128	0.159	0.112	0.142
MXL-N WTP	6	0.147	0.215	0.127	0.205	0.116	0.188	0.114	0.149	0.098	0.138
LML-Poly	12	0.185	0.254	0.191	0.266	0.144	0.235	0.136	0.194	0.113	0.152
	24	0.192	0.248	0.136	0.219	0.140	0.194	0.105	0.131	0.097	0.118
	36	0.178	0.303	0.167	0.284	0.121	0.244	0.104	0.126	0.087	0.110
	48	0.164	0.256	0.131	0.223	0.106	0.196	0.098	0.116	0.086	0.106
LML-Step	12	0.156	0.237	0.167	0.220	0.136	0.230	0.144	0.186	0.100	0.148
	24	0.159	0.256	0.195	0.253	0.144	0.205	0.117	0.142	0.085	0.138
	36	0.168	0.219	0.168	0.200	0.131	0.175	0.101	0.131	0.088	0.115
	48	0.161	0.221	0.146	0.219	0.119	0.194	0.097	0.124	0.082	0.109
LML-Spline	12	0.167	0.235	0.208	0.257	0.124	0.167	0.121	0.178	0.112	0.149
	24	0.168	0.258	0.129	0.221	0.107	0.164	0.111	0.140	0.096	0.104
	36	0.185	0.284	0.178	0.248	0.132	0.230	0.104	0.136	0.092	0.114
	48	0.194	0.270	0.194	0.264	0.169	0.164	0.100	0.137	0.086	0.098

Table A13. Means and st. dev. of modal estimates of ω_n^1 in DGP 1 (bimodal, $T = 4$)

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**
Real	–	0.55	1.26	0.55	1.26	0.55	1.26	0.55	1.26	0.55	1.26
MXL−N Pref.	6	0.89	–	0.84	–	0.75	–	0.86	–	0.80	–
		(0.11)	–	(0.14)	–	(0.10)	–	(0.15)	–	(0.16)	–
MXL−N WTP	6	0.99	–	1.02	–	1.11	–	0.94	–	0.84	–
		(0.14)	–	(0.18)	–	(0.19)	–	(0.21)	–	(0.17)	–
LML-Poly	12	0.68	2.06	0.71	1.99	0.74	1.77	0.65	1.56	0.42	1.36
		(0.07)	(0.24)	(0.06)	(0.21)	(0.08)	(0.16)	(0.05)	(0.15)	(0.03)	(0.12)
	24	0.71	2.11	0.68	2.08	0.74	1.63	0.63	1.44	0.60	1.34
		(0.07)	(0.31)	(0.06)	(0.22)	(0.06)	(0.15)	(0.06)	(0.20)	(0.03)	(0.10)
	36	0.69	2.07	0.61	1.99	0.61	1.69	0.47	1.49	0.55	1.25
		(0.05)	(0.20)	(0.06)	(0.19)	(0.04)	(0.18)	(0.04)	(0.17)	(0.03)	(0.11)
	48	0.74	2.08	0.66	2.02	0.62	1.66	0.49	1.45	0.52	1.22
		(0.06)	(0.22)	(0.05)	(0.21)	(0.07)	(0.14)	(0.04)	(0.14)	(0.02)	(0.09)
LML-Step	12	0.77	2.05	0.74	2.03	0.75	1.80	0.67	1.45	0.44	1.35
		(0.07)	(0.22)	(0.06)	(0.21)	(0.09)	(0.19)	(0.05)	(0.18)	(0.05)	(0.14)
	24	0.79	2.09	0.72	2.05	0.71	1.67	0.62	1.40	0.49	1.39
		(0.07)	(0.26)	(0.05)	(0.27)	(0.08)	(0.16)	(0.03)	(0.16)	(0.03)	(0.11)
	36	0.76	2.01	0.61	1.92	0.61	1.50	0.49	1.37	0.53	1.22
		(0.07)	(0.25)	(0.09)	(0.21)	(0.07)	(0.18)	(0.04)	(0.18)	(0.04)	(0.09)
	48	0.71	2.06	0.64	1.93	0.60	1.53	0.48	1.32	0.54	1.29
		(0.06)	(0.19)	(0.06)	(0.22)	(0.06)	(0.16)	(0.06)	(0.14)	(0.03)	(0.08)
LML-Spline	12	0.78	2.13	0.63	2.10	0.66	1.52	0.68	1.44	0.53	1.32
		(0.09)	(0.26)	(0.07)	(0.20)	(0.08)	(0.21)	(0.06)	(0.18)	(0.05)	(0.11)
	24	0.76	2.19	0.65	2.06	0.72	1.56	0.67	1.39	0.54	1.31
		(0.08)	(0.21)	(0.08)	(0.23)	(0.08)	(0.20)	(0.06)	(0.19)	(0.05)	(0.12)
	36	0.76	2.13	0.67	2.08	0.65	1.49	0.48	1.34	0.58	1.24
		(0.07)	(0.32)	(0.08)	(0.24)	(0.08)	(0.19)	(0.06)	(0.20)	(0.05)	(0.08)
	48	0.75	2.18	0.71	2.08	0.62	1.44	0.46	1.31	0.53	1.21
		(0.07)	(0.24)	(0.05)	(0.21)	(0.08)	(0.19)	(0.06)	(0.17)	(0.05)	(0.08)

Table A14. Means and st. dev. of modal estimates of ω_n^1 in DGP 1 (bimodal, $T = 8$)

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**
Real	–	0.55	1.26	0.55	1.26	0.55	1.26	0.55	1.26	0.55	1.26
MXL−N Pref.	6	0.88	–	0.80	–	0.77	–	0.74	–	0.71	–
		(0.14)	–	(0.12)	–	(0.12)	–	(0.18)	–	(0.15)	–
MXL−N WTP	6	0.95	–	0.93	–	0.91		0.94	–	0.88	–
		(0.20)	–	(0.15)	–	(0.12)	–	(0.18)	–	(0.14)	–
LML-Poly	12	0.71	2.14	0.74	2.07	0.67	1.65	0.67	1.48	0.64	1.43
		(0.06)	(0.22)	(0.06)	(0.15)	(0.07)	(0.18)	(0.05)	(0.14)	(0.04)	(0.11)
	24	0.74	2.19	0.71	2.16	0.66	1.51	0.65	1.47	0.62	1.41
		(0.06)	(0.21)	(0.07)	(0.19)	(0.05)	(0.18)	(0.06)	(0.17)	(0.06)	(0.12)
	36	0.72	2.15	0.73	2.07	0.73	1.57	0.69	1.42	0.56	1.32
		(0.07)	(0.19)	(0.05)	(0.17)	(0.06)	(0.17)	(0.04)	(0.15)	(0.05)	(0.09)
	48	0.77	2.16	0.69	2.10	0.64	1.54	0.61	1.39	0.54	1.29
		(0.05)	(0.18)	(0.06)	(0.16)	(0.05)	(0.15)	(0.04)	(0.13)	(0.04)	(0.08)
LML-Step	12	0.70	2.13	0.77	2.11	0.77	1.78	0.69	1.42	0.66	1.42
		(0.11)	(0.22)	(0.14)	(0.22)	(0.12)	(0.19)	(0.09)	(0.12)	(0.05)	(0.12)
	24	0.72	2.17	0.75	2.13	0.73	1.75	0.64	1.37	0.51	1.37
		(0.10)	(0.24)	(0.09)	(0.22)	(0.08)	(0.15)	(0.09)	(0.11)	(0.06)	(0.10)
	36	0.69	2.08	0.73	2.00	0.63	1.78	0.61	1.39	0.55	1.29
		(0.12)	(0.22)	(0.09)	(0.21)	(0.08)	(0.14)	(0.08)	(0.12)	(0.05)	(0.09)
	48	0.73	2.14	0.67	2.01	0.64	1.71	0.60	1.36	0.54	1.26
		(0.09)	(0.19)	(0.06)	(0.22)	(0.07)	(0.15)	(0.06)	(0.11)	(0.04)	(0.08)
LML-Spline	12	0.71	2.22	0.66	2.18	0.68	1.80	0.60	1.44	0.63	1.44
		(0.09)	(0.22)	(0.08)	(0.20)	(0.08)	(0.21)	(0.05)	(0.20)	(0.05)	(0.15)
	24	0.69	2.28	0.77	2.14	0.74	1.84	0.69	1.48	0.59	1.48
		(0.08)	(0.24)	(0.09)	(0.19)	(0.08)	(0.20)	(0.06)	(0.18)	(0.06)	(0.11)
	36	0.69	2.22	0.72	2.16	0.67	1.70	0.60	1.33	0.57	1.31
		(0.06)	(0.29)	(0.10)	(0.24)	(0.09)	(0.17)	(0.06)	(0.21)	(0.05)	(0.08)
	48	0.62	2.27	0.74	2.16	0.64	1.75	0.58	1.38	0.55	1.28
		(0.05)	(0.16)	(0.07)	(0.21)	(0.06)	(0.15)	(0.04)	(0.10)	(0.03)	(0.07)

Table A15. Means and st. dev. of modal estimates of ω_n^2 in DGP 1 (bimodal, $T = 4$)

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**
Real	–	−1.78	1.69	−1.78	1.69	−1.78	1.69	−1.78	1.69	−1.78	1.69
MXL−N Pref.	6	0.78	–	0.76	–	0.65	–	0.56	–	0.46	–
		(0.16)	–	(0.15)	–	(0.11)	–	(0.12)	–	(0.09)	–
MXL−N WTP	6	0.55	–	0.43	–	0.38		0.29	–	0.18	–
		(0.18)	–	(0.11)	–	(0.10)	–	(0.08)	–	(0.04)	–
LML-Poly	12	−1.14	1.22	−1.19	1.18	−1.18	1.41	−1.22	1.43	−1.23	1.50
		(0.16)	(0.25)	(0.14)	(0.22)	(0.14)	(0.16)	(0.15)	(0.19)	(0.13)	(0.15)
	24	−1.19	1.35	−1.20	1.26	−1.34	1.31	−1.37	1.37	−1.49	1.40
		(0.18)	(0.24)	(0.12)	(0.21)	(0.12)	(0.17)	(0.12)	(0.16)	(0.12)	(0.12)
	36	−1.29	1.42	−1.28	1.45	−1.40	1.44	−1.41	1.54	−1.51	1.55
		(0.11)	(0.20)	(0.12)	(0.18)	(0.11)	(0.12)	(0.17)	(0.16)	(0.15)	(0.16)
	48	−1.38	1.43	−1.45	1.51	−1.53	1.52	−1.49	1.54	−1.64	1.62
		(0.11)	(0.14)	(0.11)	(0.14)	(0.09)	(0.11)	(0.08)	(0.11)	(0.08)	(0.07)
LML-Step	12	−1.20	1.49	−1.16	1.46	−1.26	1.12	−1.33	1.15	−1.43	1.20
		(0.14)	(0.25)	(0.15)	(0.20)	(0.14)	(0.19)	(0.15)	(0.15)	(0.14)	(0.16)
	24	−1.37	1.49	−1.43	1.57	−1.37	1.34	−1.46	1.44	−1.50	1.42
		(0.16)	(0.21)	(0.13)	(0.13)	(0.17)	(0.18)	(0.14)	(0.12)	(0.13)	(0.14)
	36	−1.40	1.36	−1.46	1.40	−1.50	1.45	−1.59	1.46	−1.67	1.51
		(0.14)	(0.17)	(0.12)	(0.12)	(0.16)	(0.13)	(0.17)	(0.09)	(0.12)	(0.11)
	48	−1.26	1.11	−1.32	1.14	−1.35	1.49	−1.42	1.61	−1.66	1.73
		(0.15)	(0.15)	(0.16)	(0.15)	(0.13)	(0.12)	(0.09)	(0.09)	(0.08)	(0.06)
LML-Spline	12	−1.28	1.19	−1.23	1.12	−1.39	1.21	−1.42	1.33	−1.46	1.41
		(0.15)	(0.22)	(0.17)	(0.18)	(0.15)	(0.16)	(0.15)	(0.19)	(0.12)	(0.14)
	24	−1.28	1.19	−1.23	1.17	−1.51	1.22	−1.60	1.38	−1.54	1.40
		(0.15)	(0.18)	(0.15)	(0.21)	(0.12)	(0.16)	(0.14)	(0.16)	(0.13)	(0.16)
	36	−1.36	1.03	−1.42	1.16	−1.33	1.55	−1.40	1.59	−1.54	1.66
		(0.16)	(0.20)	(0.18)	(0.19)	(0.1)	(0.16)	(0.12)	(0.11)	(0.10)	(0.12)
	48	−1.32	1.12	−1.31	1.17	−1.43	1.59	−1.55	1.53	−1.66	1.70
		(0.16)	(0.16)	(0.14)	(0.15)	(0.09)	(0.10)	(0.08)	(0.12)	(0.07)	(0.09)

Table A16. Means and st. dev. of modal estimates of ω_n^2 in DGP 1 (bimodal, $T = 8$)

Model		$N = 70$		$N = 210$		$N = 490$		$N = 980$		$N = 1,960$	
	κ	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**	**Max1**	**Max2**
Real	–	−1.78	1.69	−1.78	1.69	−1.78	1.69	−1.78	1.69	−1.78	1.69
MXL−N Pref.	6	0.22	–	0.34	–	0.27	–	0.49	–	0.45	–
		(0.04)	–	(0.05)	–	(0.02)	–	(0.08)	–	(0.05)	–
MXL−N WTP	6	0.12	–	0.12	–	0.17		0.27	–	0.08	–
		(0.03)	–	(0.05)	–	(0.04)	–	(0.06)	–	(0.03)	–
LML-Poly	12	−1.19	1.27	−1.24	1.23	−1.23	1.47	−1.27	1.49	−1.28	1.56
		(0.19)	(0.25)	(0.16)	(0.20)	(0.15)	(0.18)	(0.16)	(0.15)	(0.12)	(0.14)
	24	−1.24	1.40	−1.25	1.31	−1.39	1.36	−1.42	1.42	−1.55	1.46
		(0.16)	(0.22)	(0.11)	(0.18)	(0.14)	(0.15)	(0.14)	(0.14)	(0.11)	(0.12)
	36	−1.34	1.48	−1.33	1.51	−1.46	1.50	−1.47	1.60	−1.57	1.61
		(0.11)	(0.21)	(0.14)	(0.15)	(0.12)	(0.13)	(0.15)	(0.13)	(0.10)	(0.13)
	48	−1.44	1.49	−1.51	1.57	−1.59	1.58	−1.55	1.60	−1.71	1.68
		(0.13)	(0.12)	(0.10)	(0.13)	(0.09)	(0.10)	(0.09)	(0.10)	(0.06)	(0.08)
LML-Step	12	−1.25	1.55	−1.21	1.52	−1.31	1.16	−1.38	1.20	−1.49	1.25
		(0.20)	(0.25)	(0.13)	(0.12)	(0.14)	(0.19)	(0.14)	(0.12)	(0.14)	(0.16)
	24	−1.42	1.55	−1.49	1.63	−1.42	1.39	−1.52	1.50	−1.56	1.48
		(0.11)	(0.21)	(0.13)	(0.13)	(0.17)	(0.18)	(0.14)	(0.12)	(0.12)	(0.16)
	36	−1.46	1.41	−1.52	1.46	−1.56	1.51	−1.65	1.52	−1.74	1.57
		(0.13)	(0.17)	(0.12)	(0.12)	(0.18)	(0.13)	(0.17)	(0.09)	(0.12)	(0.14)
	48	−1.31	1.15	−1.37	1.19	−1.40	1.55	−1.48	1.67	−1.73	1.80
		(0.12)	(0.13)	(0.15)	(0.15)	(0.11)	(0.10)	(0.09)	(0.11)	(0.06)	(0.10)
LML-Spline	12	−1.33	1.24	−1.28	1.16	−1.45	1.26	−1.48	1.38	−1.52	1.47
		(0.18)	(0.22)	(0.19)	(0.18)	(0.15)	(0.16)	(0.15)	(0.19)	(0.12)	(0.14)
	24	−1.37	1.28	−1.24	1.22	−1.57	1.27	−1.66	1.44	−1.60	1.46
		(0.15)	(0.18)	(0.15)	(0.18)	(0.12)	(0.16)	(0.14)	(0.16)	(0.13)	(0.16)
	36	−1.41	1.07	−1.48	1.21	−1.38	1.61	−1.46	1.65	−1.60	1.73
		(0.16)	(0.17)	(0.11)	(0.19)	(0.1)	(0.19)	(0.12)	(0.11)	(0.10)	(0.12)
	48	−1.37	1.16	−1.36	1.22	−1.49	1.65	−1.61	1.59	−1.73	1.77
		(0.14)	(0.11)	(0.12)	(0.11)	(0.09)	(0.09)	(0.07)	(0.12)	(0.07)	(0.06)

Table A17. Means and st. dev. of modal estimates of ω_n^1 in DGP 2 (trimodal, $T = 4$)

Model		$N = 70$			$N = 210$			$N = 490$			$N = 980$			$N = 1,960$		
	κ	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**
Real		−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61
MXL−N Pref.	6	1.13	–	–	1.34	–	–	1.27	–	–	1.19	–	–	1.25	–	–
		(0.26)			(0.29)			(0.28)			(0.23)			(0.22)		
MXL−N WTP	6	1.15	–	–	1.13	–	–	1.18	–	–	1.27	–	–	1.38	–	–
		(0.26)			(0.19)			(0.27)			(0.28)			(0.20)		
LML-Poly	12	−1.89	3.10	–	−1.80	3.16	–	−1.76	3.23	–	−1.72	3.29	–	−1.38	3.36	–
		(0.21)	(0.35)		(0.22)	(0.18)		(0.19)	(0.29)		(0.15)	(0.26)		(0.13)	(0.25)	
	24	−1.91	3.19	–	−1.81	3.25	–	−1.78	3.32	–	−1.74	2.22	4.04	−1.39	2.00	3.86
		(0.22)	(0.37)		(0.24)	(0.19)		(0.22)	(0.25)		(0.14)	(0.28)	(0.39)	(0.15)	(0.25)	(0.32)
	36	−1.86	3.32	–	−1.77	2.24	4.37	−1.73	2.20	4.28	−1.70	2.15	4.20	−1.36	1.94	3.72
		(0.19)	(0.32)		(0.21)	(0.25)	(0.41)	(0.24)	(0.24)	(0.38)	(0.13)	(0.21)	(0.35)	(0.16)	(0.22)	(0.34)
	48	−1.77	2.33	4.39	−1.68	2.21	4.17	−1.79	2.17	4.09	−1.61	2.13	4.01	−1.29	1.91	3.68
		(0.18)	(0.28)	(0.46)	(0.19)	(0.26)	(0.38)	(0.22)	(0.21)	(0.36)	(0.10)	(0.20)	(0.33)	(0.12)	(0.18)	(0.29)
LML-Step	12	−1.98	3.26	–	−1.88	3.32	–	−1.84	3.39	–	−1.81	3.45	–	−1.45	3.52	–
		(0.24)	(0.31)		(0.21)	(0.22)		(0.22)	(0.19)		(0.12)	(0.29)		(0.12)	(0.17)	
	24	−1.90	3.35	–	−1.80	3.42	–	−1.76	3.48	–	−1.73	2.19	4.15	−1.38	1.97	3.82
		(0.23)	(0.27)		(0.20)	(0.22)		(0.23)	(0.18)		(0.15)	(0.31)	(0.46)	(0.13)	(0.16)	(0.33)
	36	−1.83	3.49	–	−1.74	2.26	4.20	−1.70	2.22	4.37	−1.67	2.17	4.03	−1.34	1.95	3.71
		(0.25)	(0.26)		(0.24)	(0.25)	(0.42)	(0.27)	(0.16)	(0.39)	(0.14)	(0.26)	(0.44)	(0.15)	(0.14)	(0.36)
	48	−1.76	2.42	4.46	−1.67	2.30	4.24	−1.77	2.25	4.24	−1.61	2.21	4.07	−1.28	1.89	3.68
		(0.21)	(0.22)	(0.51)	(0.21)	(0.23)	(0.40)	(0.19)	(0.20)	(0.37)	(0.14)	(0.20)	(0.39)	(0.09)	(0.15)	(0.34)
LML-Spline	12	−2.01	3.18	–	−1.91	3.24	–	−1.87	3.31	–	−1.83	3.37	–	−1.47	3.44	–
		(0.14)	(0.27)		(0.21)	(0.19)		(0.22)	(0.21)		(0.19)	(0.15)		(0.16)	(0.25)	
	24	−1.89	3.27	–	−1.90	3.34	–	−1.76	3.40	–	−1.75	2.15	4.11	−1.38	2.01	3.79
		(0.15)	(0.29)		(0.19)	(0.24)		(0.22)	(0.23)		(0.18)	(0.25)	(0.49)	(0.14)	(0.29)	(0.40)
	36	−1.89	3.40	–	−1.80	3.19	–	−1.79	3.15	–	−1.72	2.11	4.09	−1.35	1.93	3.76
		(0.12)	(0.32)		(0.24)	(0.15)		(0.22)	(0.24)		(0.17)	(0.26)	(0.42)	(0.16)	(0.28)	(0.42)
	48	−1.83	2.39	4.40	−1.74	2.27	4.18	−1.70	2.23	4.10	−1.71	2.18	4.01	−1.34	1.92	3.69
		(0.16)	(0.24)	(0.44)	(0.20)	(0.13)	(0.41)	(0.22)	(0.17)	(0.22)	(0.11)	(0.23)	(0.38)	(0.11)	(0.24)	(0.36)

Table A18. Means and st. dev. of modal estimates of ω_n^1 in DGP 2 (trimodal, $T = 8$)

Model		$N = 70$			$N = 210$			$N = 490$			$N = 980$			$N = 1,960$		
	κ	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**
Real		−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61	−1.12	1.77	3.61
MXL−N Pref.	6	1.24	–	–	1.28	–	–	1.35	–	–	1.38	–	–	1.43	–	–
		(0.22)			(0.23)			(0.25)			(0.23)			(0.28)		
MXL−N WTP	6	1.26	–	–	1.33	–	–	1.38	–	–	1.57	–	–	1.49	–	–
		(0.27)			(0.30)			(0.25)			(0.26)			(0.22)		
LML-Poly	12	−1.80	2.95	–	−1.71	3.00	–	−1.67	3.06	–	−1.64	3.13	–	−1.31	3.19	–
		(0.23)	(0.33)		(0.21)	(0.19)		(0.20)	(0.31)		(0.13)	(0.26)		(0.14)	(0.22)	
	24	−1.81	3.03	–	−1.72	3.09	–	−1.69	3.15	4.18	−1.66	2.11	3.84	−1.32	1.90	3.67
		(0.25)	(0.35)		(0.24)	(0.21)		(0.23)	(0.23)	(0.41)	(0.12)	(0.28)	(0.37)	(0.13)	(0.26)	(0.35)
	36	−1.77	3.15	–	−1.68	2.13	4.15	−1.65	2.09	4.07	−1.61	2.05	3.99	−1.29	1.84	3.53
		(0.20)	(0.30)		(0.21)	(0.24)	(0.42)	(0.22)	(0.22)	(0.36)	(0.12)	(0.21)	(0.36)	(0.14)	(0.24)	(0.33)
	48	−1.68	2.21	4.17	−1.60	2.10	3.96	−1.57	2.06	3.88	−1.53	2.02	3.81	−1.23	1.82	3.50
		(0.16)	(0.22)	(0.44)	(0.20)	(0.24)	(0.37)	(0.21)	(0.19)	(0.38)	(0.12)	(0.20)	(0.32)	(0.11)	(0.15)	(0.28)
LML-Step	12	−1.88	3.09	–	−1.79	3.15	–	−1.75	3.22	–	−1.72	3.28	–	−1.37	3.35	–
		(0.25)	(0.30)		(0.20)	(0.23)		(0.25)	(0.19)		(0.11)	(0.23)		(0.13)	(0.17)	
	24	−1.80	3.18	–	−1.71	3.25	–	−1.68	3.31	4.21	−1.64	2.08	3.94	−1.31	1.87	3.63
		(0.24)	(0.24)		(0.22)	(0.24)		(0.26)	(0.17)	(0.43)	(0.15)	(0.32)	(0.48)	(0.13)	(0.14)	(0.37)
	36	−1.74	3.31	–	−1.65	2.15	3.99	−1.62	2.10	4.16	−1.59	2.06	3.91	−1.27	1.86	3.58
		(0.22)	(0.25)		(0.25)	(0.22)	(0.45)	(0.23)	(0.18)	(0.37)	(0.14)	(0.22)	(0.43)	(0.14)	(0.14)	(0.38)
	48	−1.67	2.30	4.24	−1.68	2.18	4.03	−1.66	2.14	4.08	−1.53	2.02	3.87	−1.22	1.81	3.51
		(0.24)	(0.24)	(0.53)	(0.22)	(0.25)	(0.42)	(0.21)	(0.22)	(0.34)	(0.17)	(0.19)	(0.40)	(0.10)	(0.16)	(0.33)
LML-Spline	12	−1.91	3.02	–	−1.81	3.08	–	−1.78	3.14	–	−1.74	3.20	–	−1.39	3.27	–
		(0.16)	(0.27)		(0.21)	(0.19)		(0.22)	(0.21)		(0.19)	(0.15)		(0.16)	(0.25)	
	24	−1.80	3.11	–	−1.71	3.17	–	−1.67	3.23	4.32	−1.64	2.05	3.91	−1.31	1.84	3.60
		(0.15)	(0.29)		(0.19)	(0.24)		(0.22)	(0.23)	(0.46)	(0.18)	(0.25)	(0.49)	(0.14)	(0.29)	(0.40)
	36	−1.88	3.23	–	−1.73	2.08	4.12	−1.67	2.04	4.04	−1.66	2.00	3.88	−1.29	1.83	3.57
		(0.15)	(0.32)		(0.24)	(0.15)	(0.48)	(0.22)	(0.24)	(0.44)	(0.17)	(0.26)	(0.42)	(0.16)	(0.28)	(0.42)
	48	−1.74	2.27	4.18	−1.65	2.16	3.97	−1.62	2.11	3.96	−1.59	1.98	3.81	−1.27	1.79	3.51
		(0.15)	(0.24)	(0.44)	(0.20)	(0.13)	(0.41)	(0.22)	(0.17)	(0.22)	(0.11)	(0.23)	(0.38)	(0.11)	(0.24)	(0.36)

Table A19. Means and st. dev. of modal estimates of ω_n^2 in DGP 2 (trimodal, $T = 4$)

Model		$N=70$			$N=210$			$N=490$			$N=980$			$N=1,960$		
	κ	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**
Real		1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82
MXL−N Pref.	6	3.92	–	–	4.01	–	–	4.11	–	–	4.05	–	–	4.01	–	–
		(0.47)			(0.51)			(0.48)			(0.46)			(0.42)		
MXL−N WTP	6	4.10	–	–	4.04	–	–	3.99	–	–	4.08	–	–	4.12	–	–
		(0.58)			(0.49)			(0.47)			(0.39)			(0.36)		
LML-Poly	12	0.26	3.24	–	0.41	3.21	–	0.74	3.46	–	0.80	3.53	–	0.87	3.60	–
		(0.04)	(0.41)		(0.05)	(0.28)		(0.13)	(0.29)		(0.08)	(0.30)		(0.12)	(0.29)	
	24	0.25	3.21	–	0.24	3.60	–	0.29	3.68	–	0.31	3.28	5.25	0.34	3.38	5.34
		(0.04)	(0.36)		(0.06)	(0.31)		(0.09)	(0.32)		(0.05)	(0.29)	(0.48)	(0.03)	(0.28)	(0.45)
	36	0.29	3.29	–	0.50	3.08	5.14	0.67	3.31	5.21	0.72	3.38	5.31	0.79	3.45	5.43
		(0.03)	(0.37)		(0.11)	(0.30)	(0.55)	(0.16)	(0.26)	(0.51)	(0.11)	(0.28)	(0.49)	(0.10)	(0.28)	(0.42)
	48	0.68	3.01	5.15	0.79	3.32	5.21	0.87	3.34	5.27	0.94	3.41	5.34	1.02	3.50	5.55
		(0.13)	(0.35)	(0.53)	(0.12)	(0.29)	(0.48)	(0.19)	(0.28)	(0.46)	(0.08)	(0.25)	(0.44)	(0.08)	(0.25)	(0.39)
LML-Step	12	0.33	3.31	–	0.38	3.50	–	0.48	3.63	–	0.51	3.70	–	0.56	3.75	–
		(0.05)	(0.40)		(0.06)	(0.30)		(0.07)	(0.32)		(0.06)	(0.26)		(0.08)	(0.26)	
	24	0.15	3.32	–	0.17	3.28	–	0.17	3.24	–	0.19	3.25	5.16	0.21	3.27	5.29
		(0.02)	(0.38)		(0.03)	(0.31)		(0.01)	(0.29)		(0.02)	(0.25)	(0.46)	(0.05)	(0.30)	(0.44)
	36	0.22	3.38	–	0.43	3.12	5.18	0.69	3.19	5.20	0.75	3.30	5.22	0.81	3.37	5.25
		(0.02)	(0.35)		(0.08)	(0.28)	(0.43)	(0.10)	(0.28)	(0.47)	(0.12)	(0.27)	(0.42)	(0.10)	(0.28)	(0.45)
	48	0.78	3.04	5.22	0.82	3.22	5.25	0.85	3.29	5.23	0.92	3.36	5.31	1.00	3.45	5.49
		(0.10)	(0.32)	(0.49)	(0.11)	(0.29)	(0.45)	(0.12)	(0.25)	(0.41)	(0.15)	(0.24)	(0.40)	(0.13)	(0.22)	(0.38)
LML-Spline	12	0.40	3.36	–	0.61	3.47	–	0.91	3.69	–	0.98	3.77	–	1.07	3.86	–
		(0.06)	(0.34)		(0.25)	(0.20)		(0.24)	(0.35)		(0.16)	(0.21)		(0.13)	(0.31)	
	24	0.09	3.41	–	0.17	3.61	–	0.19	3.36	–	0.21	3.21	5.13	0.23	3.23	5.29
		(0.01)	(0.41)		(0.26)	(0.23)		(0.03)	(0.32)		(0.18)	(0.22)	(0.27)	(0.17)	(0.24)	(0.25)
	36	0.25	3.35	–	0.38	3.43	–	0.43	3.53	–	0.46	3.26	5.23	0.50	3.31	5.36
		(0.04)	(0.39)		(0.20)	(0.21)		(0.06)	(0.31)		(0.15)	(0.25)	(0.28)	(0.16)	(0.28)	(0.27)
	48	0.85	3.02	5.18	0.75	3.25	5.23	0.93	3.28	5.31	1.00	3.34	5.33	1.09	3.44	5.60
		(0.21)	(0.33)	(0.35)	(0.22)	(0.18)	(0.35)	(0.19)	(0.29)	(0.35)	(0.15)	(0.24)	(0.23)	(0.12)	(0.24)	(0.25)

Table A20. Means and st. dev. of modal estimates of ω_n^2 in DGP 2 (trimodal, $T = 8$)

Model		$N = 70$			$N = 210$			$N = 490$			$N = 980$			$N = 1,960$		
	κ	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**	**Max1**	**Max2**	**Max3**
Real		1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82	1.36	3.68	5.82
MXL−N Pref.	6	3.86	–	–	3.97	–	–	4.04	–	–	4.08	–	–	4.15	–	–
		(0.45)			(0.46)			(0.44)			(0.43)			(0.44)		
MXL−N WTP	6	3.98	–	–	3.95	–	–	4.14	–	–	4.12	–	–	4.20	–	–
		(0.50)			(0.48)			(0.42)			(0.47)			(0.42)		
LML-Poly	12	0.31	3.33	–	0.43	3.31	–	0.76	3.56	–	0.81	3.60	–	0.91	3.68	–
		(0.06)	(0.40)		(0.05)	(0.28)		(0.13)	(0.29)		(0.08)	(0.30)		(0.14)	(0.23)	
	24	0.28	3.30	–	0.25	3.71	–	0.30	3.81	–	0.32	3.39	5.31	0.35	3.40	5.35
		(0.05)	(0.35)		(0.06)	(0.31)		(0.09)	(0.32)		(0.05)	(0.26)	(0.46)	(0.09)	(0.23)	(0.44)
	36	0.29	3.33	–	0.67	3.20	5.28	0.70	3.40	5.43	0.74	3.47	5.32	0.83	3.51	5.44
		(0.05)	(0.36)		(0.11)	(0.30)	(0.55)	(0.16)	(0.26)	(0.51)	(0.11)	(0.24)	(0.52)	(0.11)	(0.25)	(0.40)
	48	0.71	3.12	5.16	0.84	3.45	5.40	0.90	3.39	5.36	0.95	3.51	5.52	1.07	3.55	5.66
		(0.13)	(0.35)	(0.53)	(0.12)	(0.29)	(0.48)	(0.19)	(0.28)	(0.46)	(0.09)	(0.27)	(0.45)	(0.09)	(0.24)	(0.37)
LML-Step	12	0.38	3.43	–	0.39	3.50	–	0.49	3.71	–	0.52	3.79	–	0.58	3.94	–
		(0.05)	(0.38)		(0.06)	(0.30)		(0.05)	(0.32)		(0.05)	(0.26)		(0.08)	(0.25)	
	24	0.18	3.48	–	0.18	3.36	–	0.18	3.32	–	0.19	3.26	5.19	0.21	3.30	5.36
		(0.04)	(0.36)		(0.07)	(0.30)		(0.06)	(0.29)		(0.04)	(0.25)	(0.46)	(0.03)	(0.30)	(0.44)
	36	0.29	3.53	–	0.66	3.21	5.27	0.71	3.31	5.45	0.78	3.39	5.29	0.82	3.49	5.43
		(0.05)	(0.35)		(0.11)	(0.26)	(0.43)	(0.12)	(0.28)	(0.45)	(0.13)	(0.27)	(0.42)	(0.10)	(0.26)	(0.45)
	48	0.82	3.17	5.31	0.82	3.24	5.46	0.86	3.33	5.35	0.93	3.39	5.36	1.04	3.59	5.69
		(0.12)	(0.29)	(0.49)	(0.14)	(0.26)	(0.45)	(0.14)	(0.25)	(0.40)	(0.16)	(0.24)	(0.40)	(0.13)	(0.22)	(0.38)
LML-Spline	12	0.41	3.46	–	0.62	3.48	–	0.94	3.70	–	1.01	3.89	–	1.08	4.05	–
		(0.07)	(0.40)		(0.15)	(0.24)		(0.13)	(0.32)		(0.16)	(0.26)		(0.18)	(0.34)	
	24	0.09	3.43	–	0.17	3.78	–	0.20	3.40	–	0.21	3.30	5.20	0.24	3.35	5.48
		(0.03)	(0.34)		(0.04)	(0.28)		(0.09)	(0.32)		(0.05)	(0.26)	(0.25)	(0.05)	(0.27)	(0.32)
	36	0.34	3.51	–	0.71	3.31	5.38	0.44	3.32	5.40	0.47	3.27	5.45	0.52	3.41	5.46
		(0.07)	(0.33)		(0.12)	(0.26)	(0.41)	(0.10)	(0.30)	(0.38)	(0.07)	(0.26)	(0.25)	(0.14)	(0.28)	(0.35)
	48	0.85	3.14	5.41	0.92	3.34	5.32	0.93	3.41	5.43	1.02	3.42	5.59	1.15	3.60	5.63
		(0.14)	(0.36)	(0.35)	(0.14)	(0.28)	(0.35)	(0.12)	(0.32)	(0.35)	(0.13)	(0.28)	(0.31)	(0.16)	(0.28)	(0.30)